Fuzzy Logic Basics

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Slides accompanying The MIT Press‘ book:: Learning and Soft Computing

The slides shown here are developed around the basic notion of

Embedding Structured Human Knowledge into workable mathematical models by Fuzzy Logic

as presented in the book

LEARNING AND SOFT COMPUTING

Support Vector Machines, Neural Networks and Fuzzy Logic Models

Author: Vojislav KECMAN
The MIT Press, Cambridge, MA, 2001
ISBN 0-262-11255-8
608 pp., 268 illustrations, 47 examples, 155 problems

They are intended to support both the instructors in the development and delivery of course content and the learners in acquiring the ideas and techniques presented in the book in a more pleasant way than just reading.

What is Fuzzy Logic ?

- FL is a Tool for Embedding Human Structured Knowledge (Experience, Expertise, Heuristic)

Why is it Fuzzy ?

- Because - human knowledge is fuzzy : expressed in 'Fuzzy' Linguistic Terms - Young, Old, Big, Cheap are FUZZY words

  Temperature is expressed as Cold, Warm or Hot. No quantitative meaning.

"Fuzzy Logic may be viewed as a bridge over the excessively wide gap between the precision of classical crisp logic and the imprecision of both the real world and its human interpretation"

Paraphrasing L. Zadeh

- FL attempts to model the way of reasoning that goes in the human brain.
- Almost all of human experience is stored in the form of the IF - THEN rules.
- Human reasoning is pervasively approximate, non-quantitative, linguistic, and dispositional (meaning, usually qualified).

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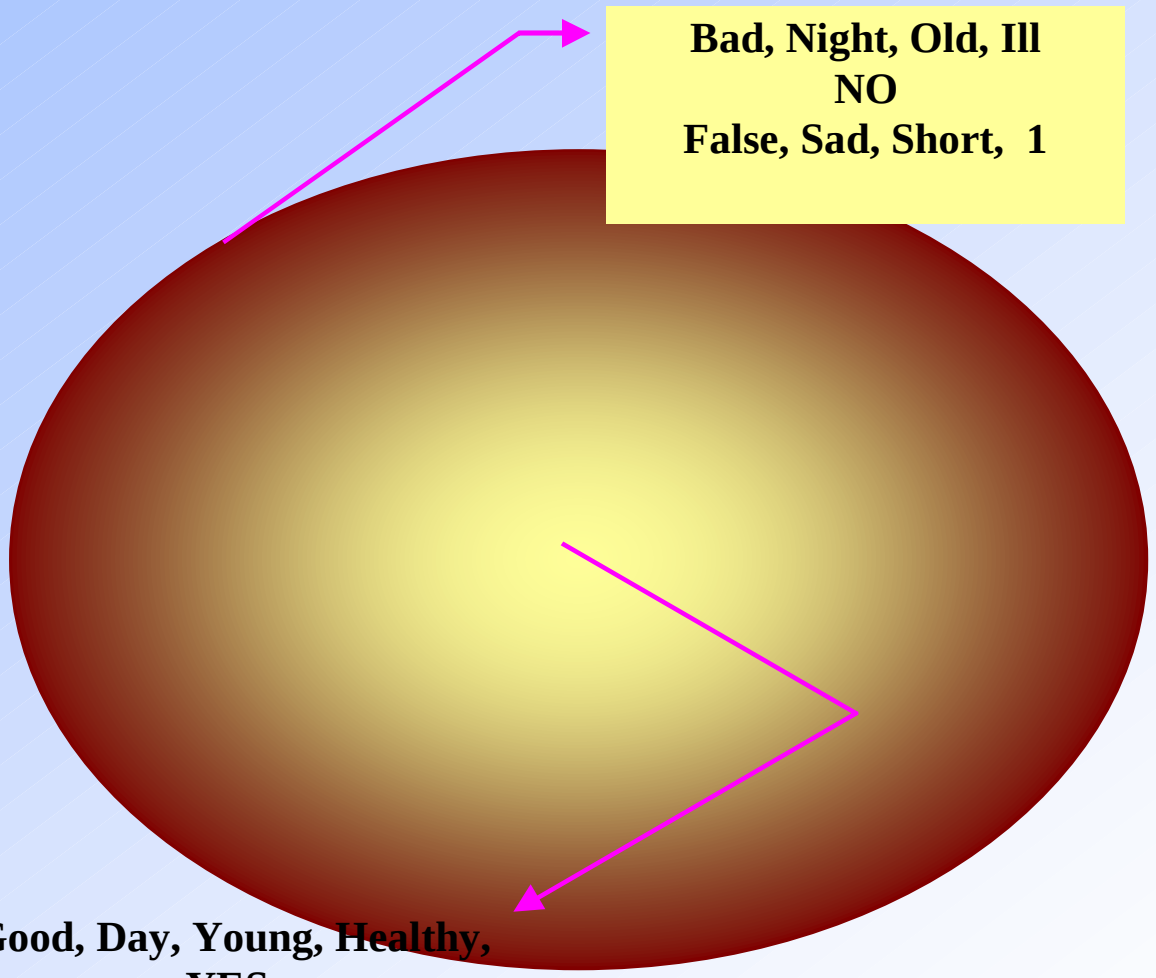
- **Why is that way ?**

The World is Not Binary!
Gradual Transitions & Ambiguities at the Boundaries



Criteria: When and Why to Apply FL

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- Human (Structured) Knowledge Available

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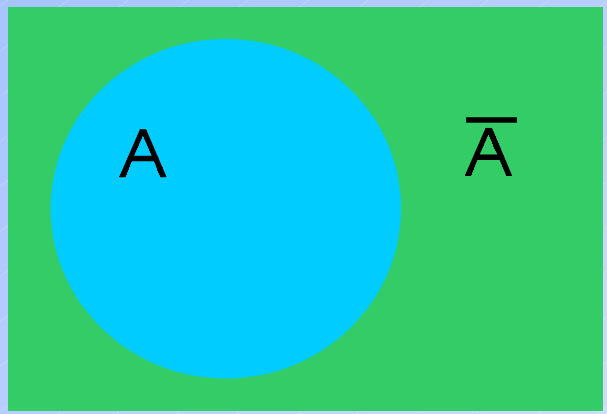
- At the Higher Levels of Hierarchical Control Systems
- In Generic Decision Making Process

How to Transfer Human Knowledge Into the Model ?

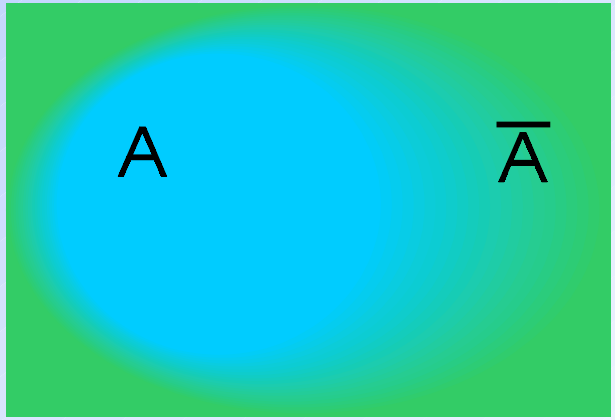
- Knowledge must be structured !
- Possible shortcomings:
  - Knowledge is very subjective category
  - 'Experts' bounce between some extreme poles:
    - Have problems with structuring the knowledge, or
    - Too aware in his/hers expertise, or
    - Tend to hide 'knowledge', or ...
- Solution: Find a Good Expert! There are always some around.

BASICS ABOUT CRISP AND FUZZY SETS

Crisp Sets

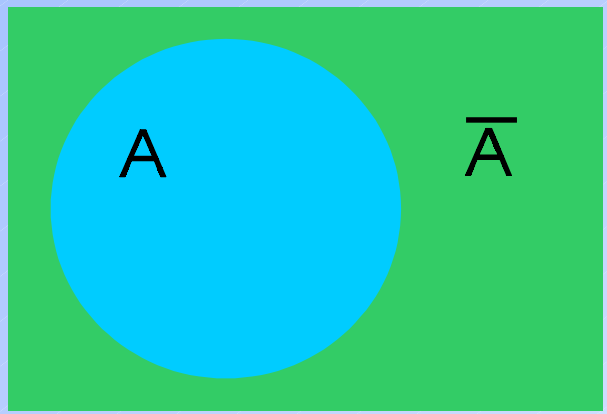


Fuzzy Sets

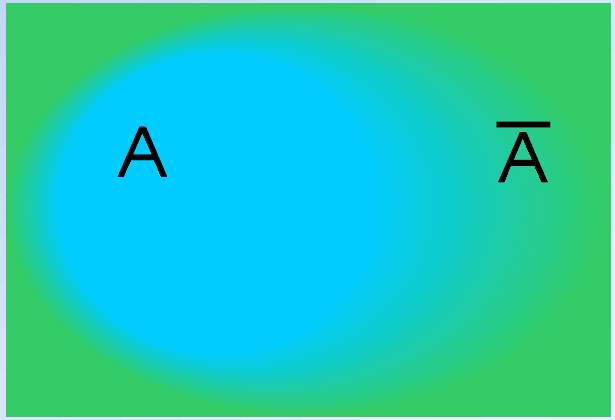


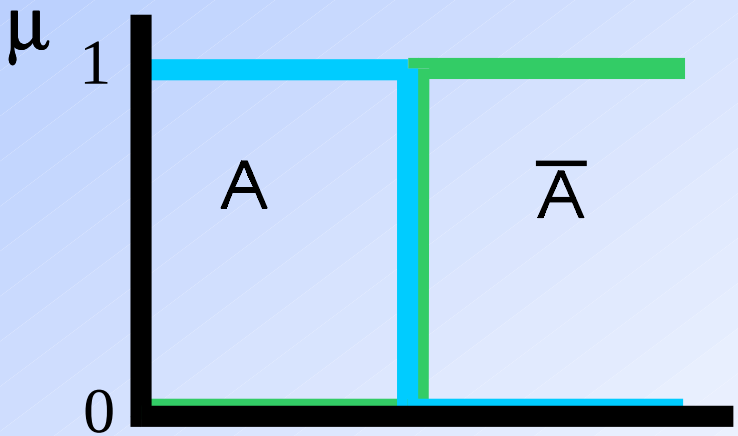
Venn Diagrams

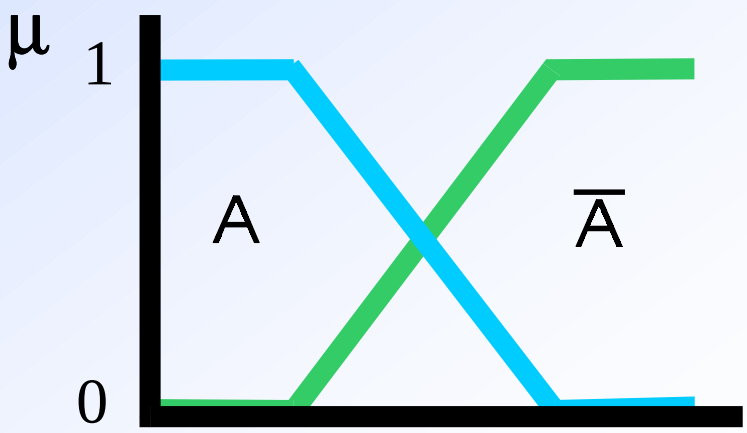
Crisp Sets



Fuzzy Sets



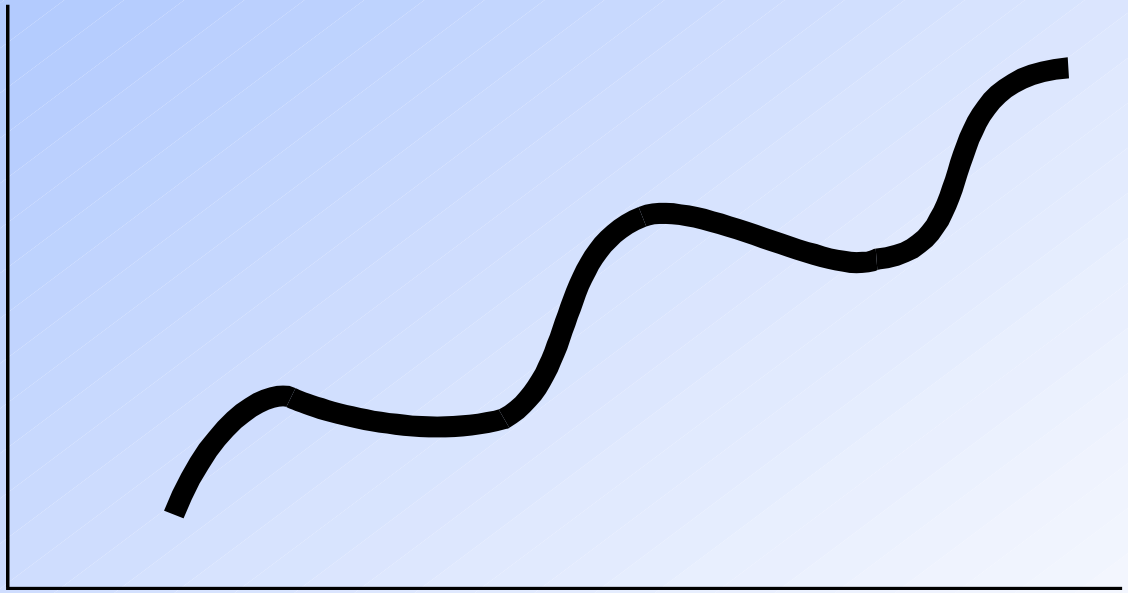




μ - membership degree, possibility distribution, grade of belonging

Modeling or Approximating A Function:

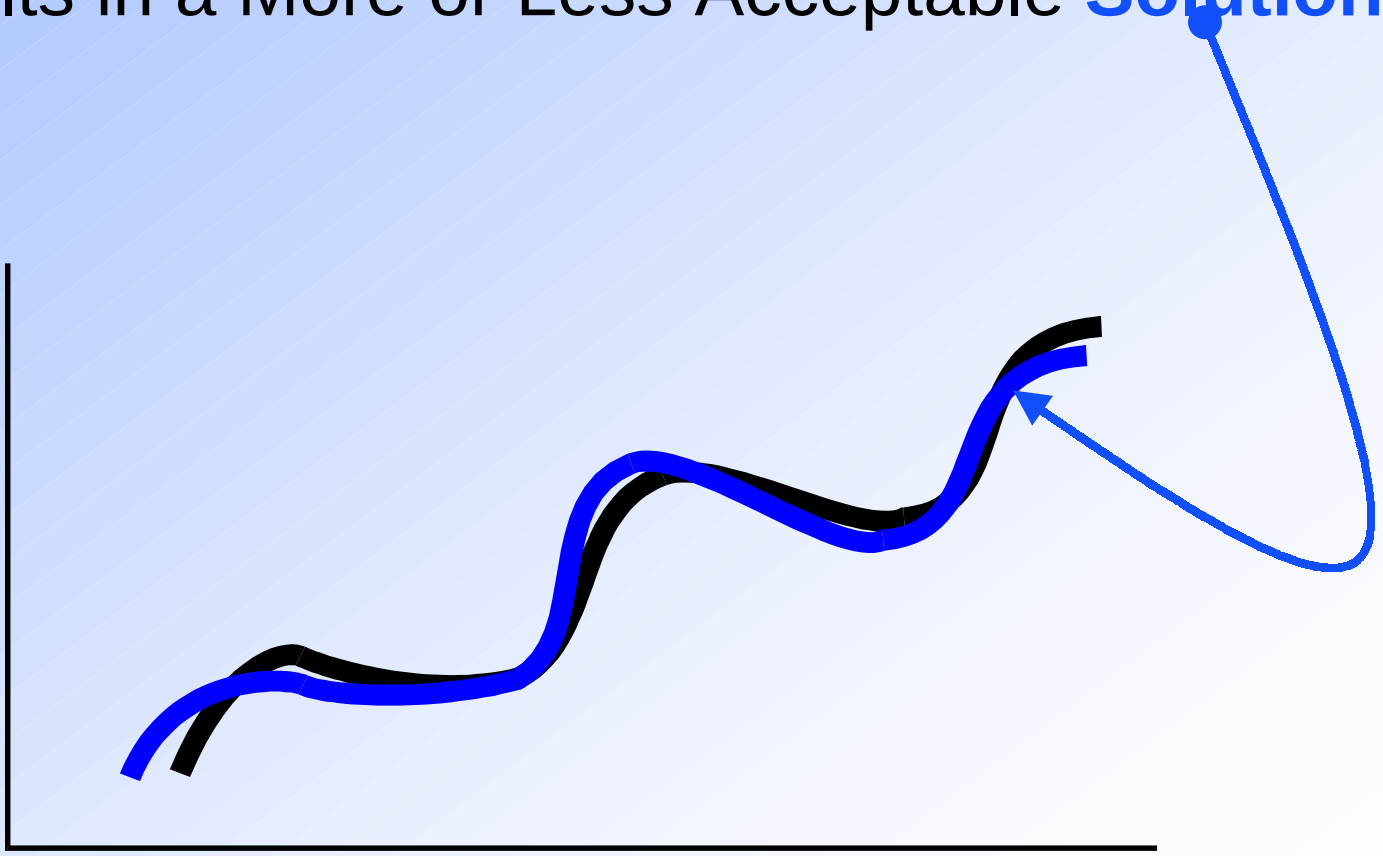
Curve or Surface Fitting



More Different Names in Different Disciplines:
Regression (L or NL), Estimation, Identification, Filtering

Modeling A Function

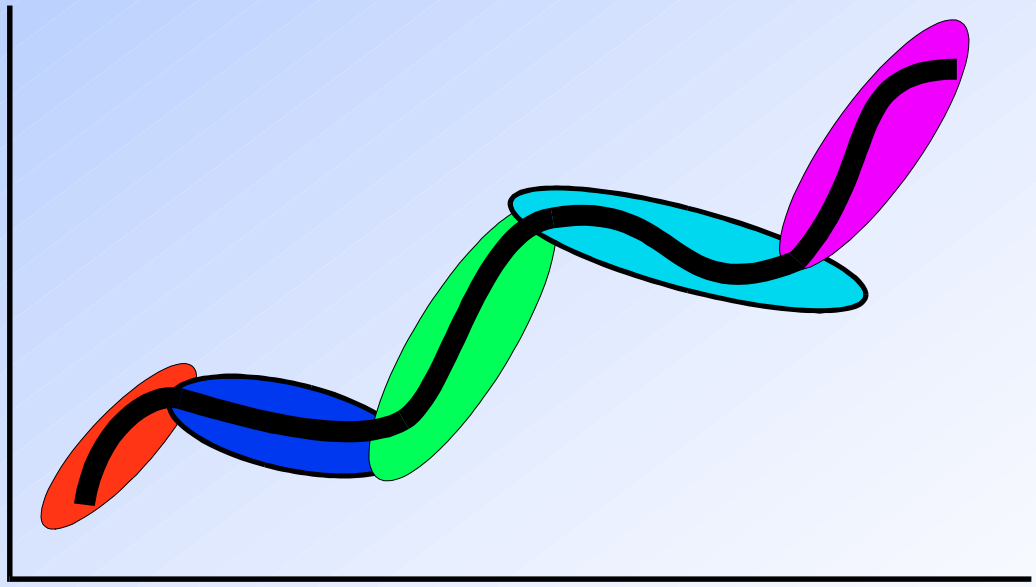
Standard Mathematical Procedure of Curve Fitting
Results in a More or Less Acceptable **Solution**



Modeling A Function

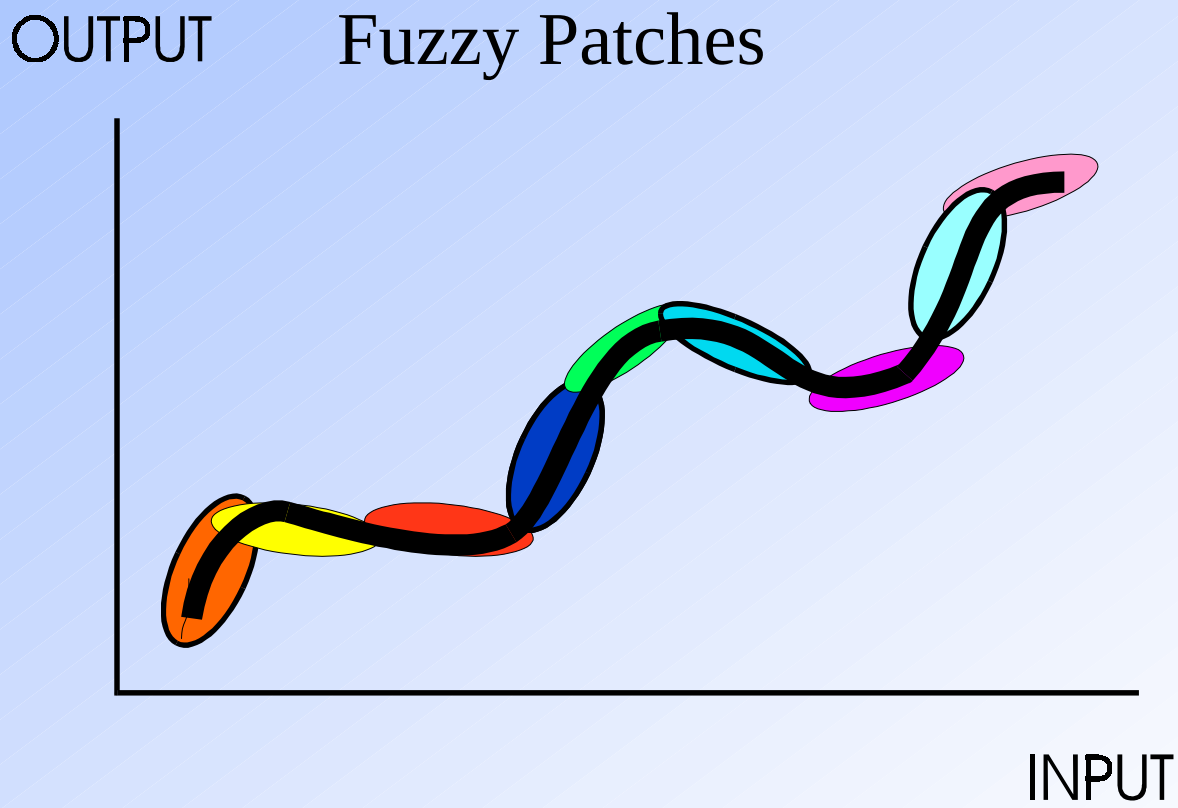
Curve Fitting by Using Fuzzy Rules (Patches)

When There Are More Inputs We Try to Approximate
A Surface (2 Inputs) or Hyper-Surface (3 or More Inputs)



Small Number of Rules - Big Patches or Rough Approximation

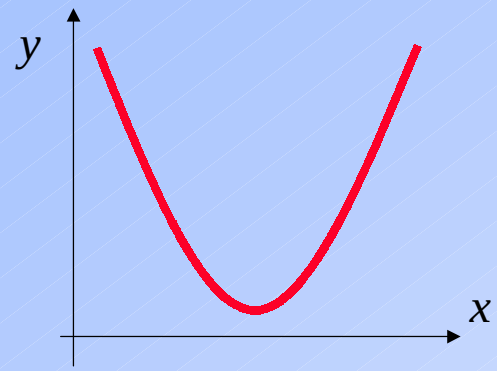
Modeling A Function

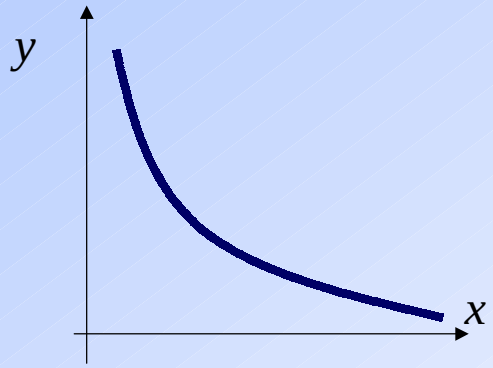


More Rules - More Smaller Patches and Better Approximation

Origin of the patches and how do they work?

Consider modeling two different functions by fuzzy rules (book's Fig 1.3.)





Less rules leads to the approximation accuracy decrease. An increase in a number of rules increases the precision at the cost of a computation time needed to process more rules. This is the most classical soft computing dilemma that trades off between the imprecision and uncertainty on one hand and low solution cost, tractability and robustness on the other. The appropriate rules for functions in Fig 1.3 are:

Left graph

IF *x* is *low* THEN *y* is *high.*
IF *x* is *medium* THEN *y* is *low.*
IF *x* is *big* THEN *y* is *high.*

Right graph

IF *x* is *low* THEN *y* is *high.*
IF *x* is *medium* THEN *y* is *medium.*
IF *x* is *big* THEN *y* is *low.*

These rules define three large rectangular patches that cover our functions. They are shown in the next slide (Fig 1.4 in the book) together with two possible approximators for each function.

Modeling two different functions by fuzzy rules (book's Fig 1.4.)

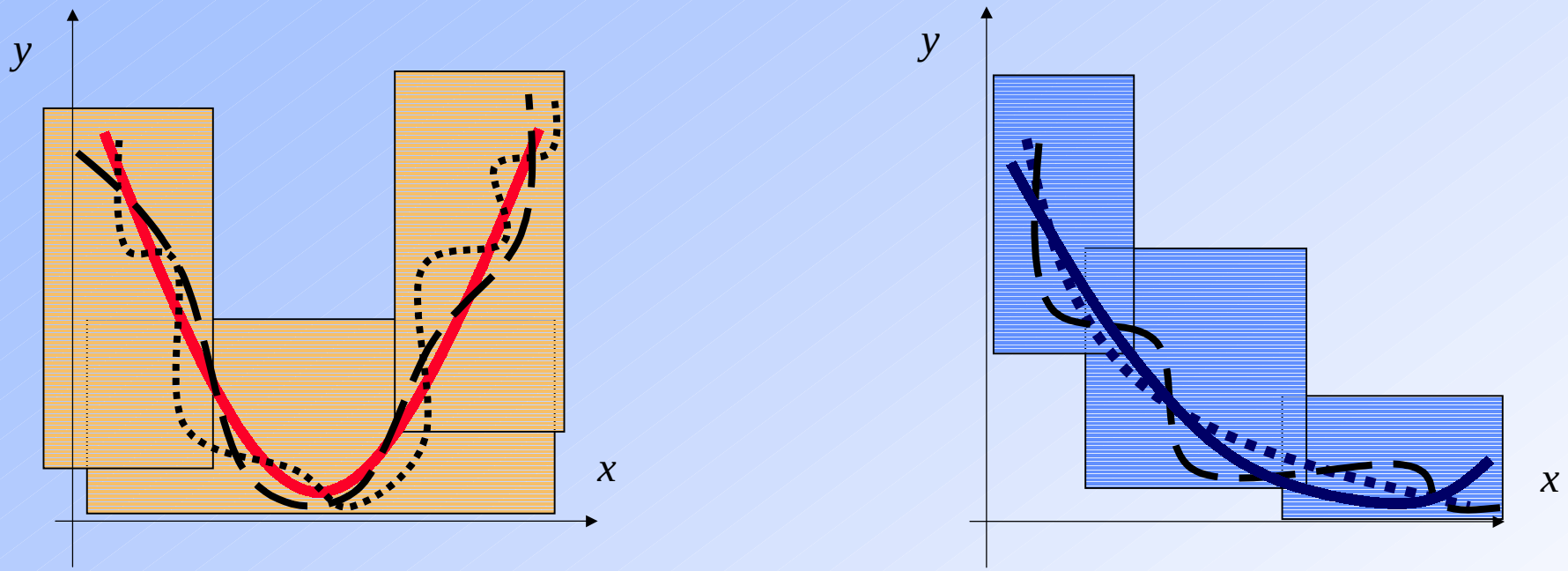


Figure 1.4 Two different **functions** (**solid lines** in both graphs) covered by three patches produced by IF-THEN rules and modeled by two possible *approximators* (*dashed* and *dotted curves*).

COMENTS: Note that humans do not (or only rarely) think in terms of nonlinear functions. We do not try to 'draw these functions in our mind'. We neither try 'to see' them as geometrical artifacts. In general, we do not process geometrical figures, curves, surfaces or hypersurfaces while performing some tasks or expressing our knowledge. Even more, our expertise or understanding of some functional dependencies is very often not a structured piece of knowledge at all. We typically perform very complex tasks without being able to express how are we executing them.

The curious reader should try, for example, to explain to his/hers colleague, in the form of IF-THEN rules, how s/he is riding the bike, recognizing numerals or surfing.

Fuzzy Control of the Distance Between Two Cars

General approach in all fuzzy modeling is always the same.

***i*) Define the variables of relevance, interest or importance:**

In engineering we use to call them Input and Output Variables,

***ii*) Define the subsets' intervals:**

Small - Medium, or Negative - Positive, or
Left - Right (labels are variables dependant)

***iii*) Choose the Shapes and the Positions of Fuzzy Subsets i.e., Membership Functions i.e., Attributes**

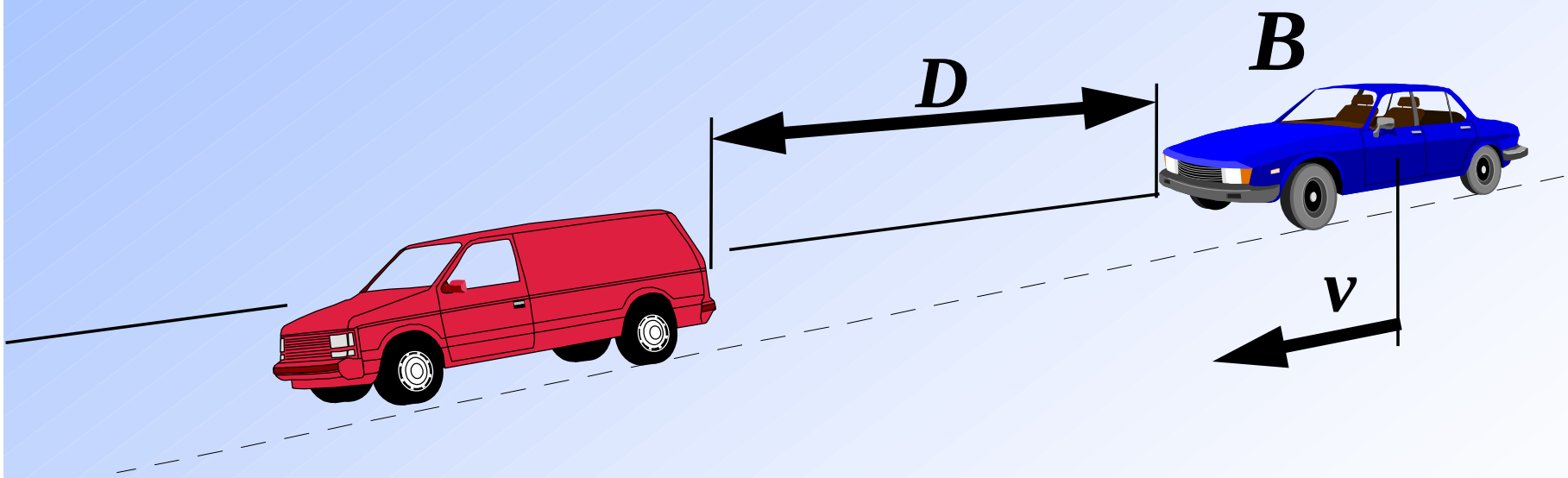
***iv*) Set the Rule Basis i.e., IF - THEN Rules:**

***v*) Perform calculations and (if needed) tune (learn, adjust, adapt) the positions and the shapes of both the input and the output attributes of the model.**

Fuzzy Control of the Distance Between Two Cars

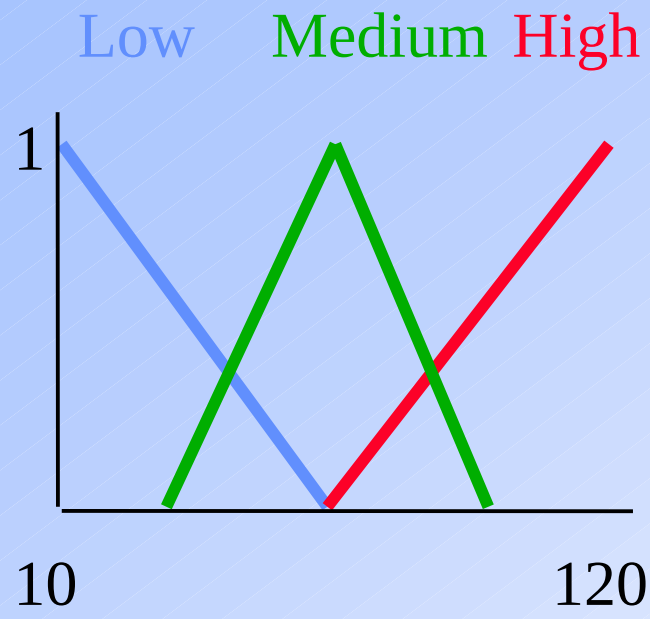
INPUTS: ***D*** = DISTANCE, ***v*** = SPEED

OUTPUT: ***B*** = BRAKING FORCE

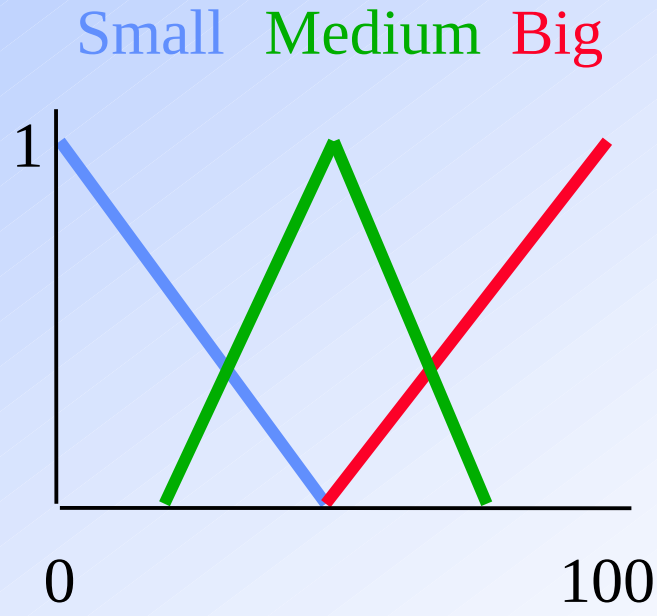


Let's analyze the rules for some <u>given distance</u> **D** and for different velocities **v** i.e., $B = f(v)$

Velocity



Braking Force

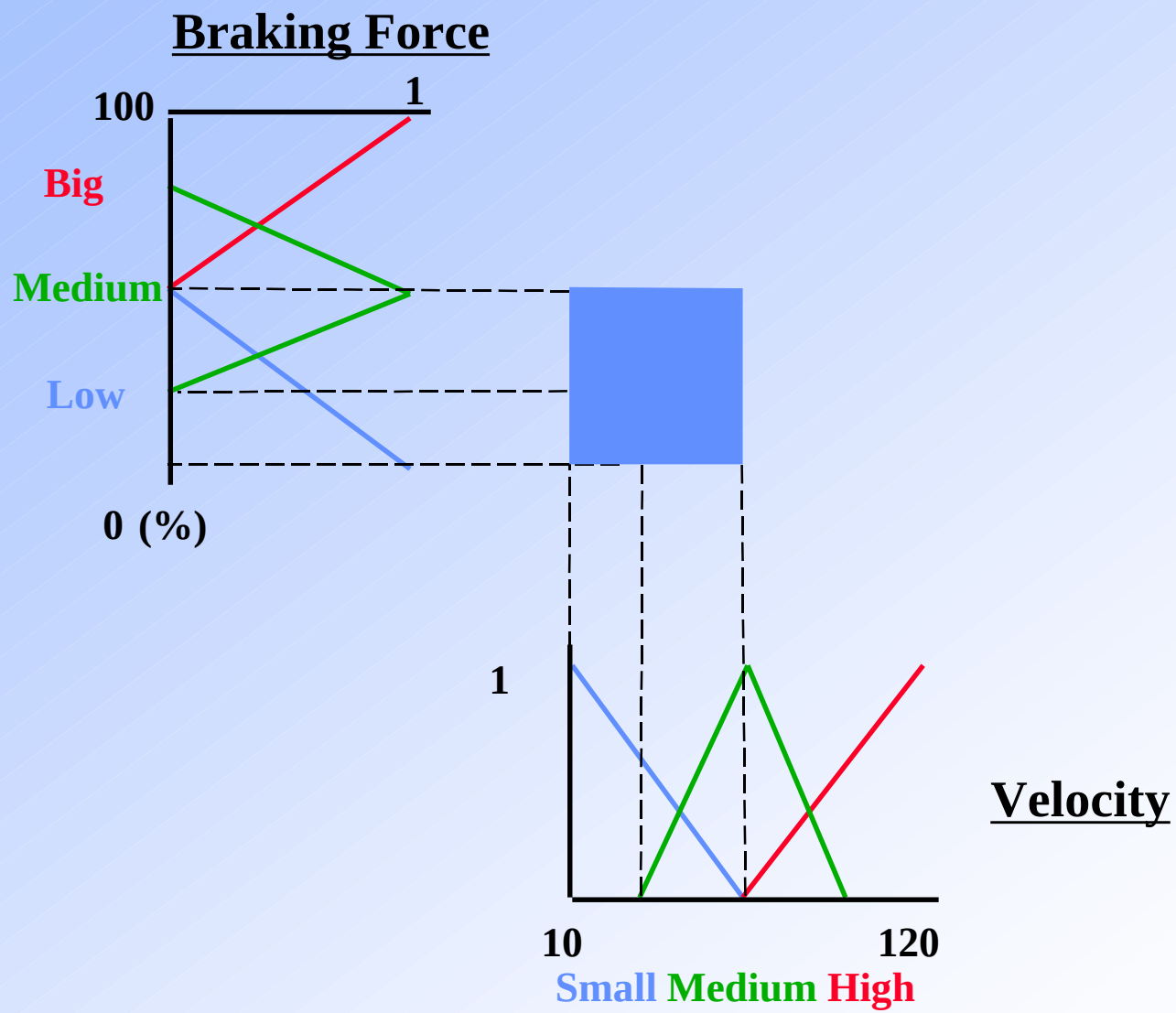


IF the Velocity is **Low, THEN** the Braking Force is **Small.**

IF the Velocity is **Medium, THEN** the Braking Force is **Medium.**

IF the Velocity is **High, THEN** the Braking Force is **Big.**

The Fuzzy Patches



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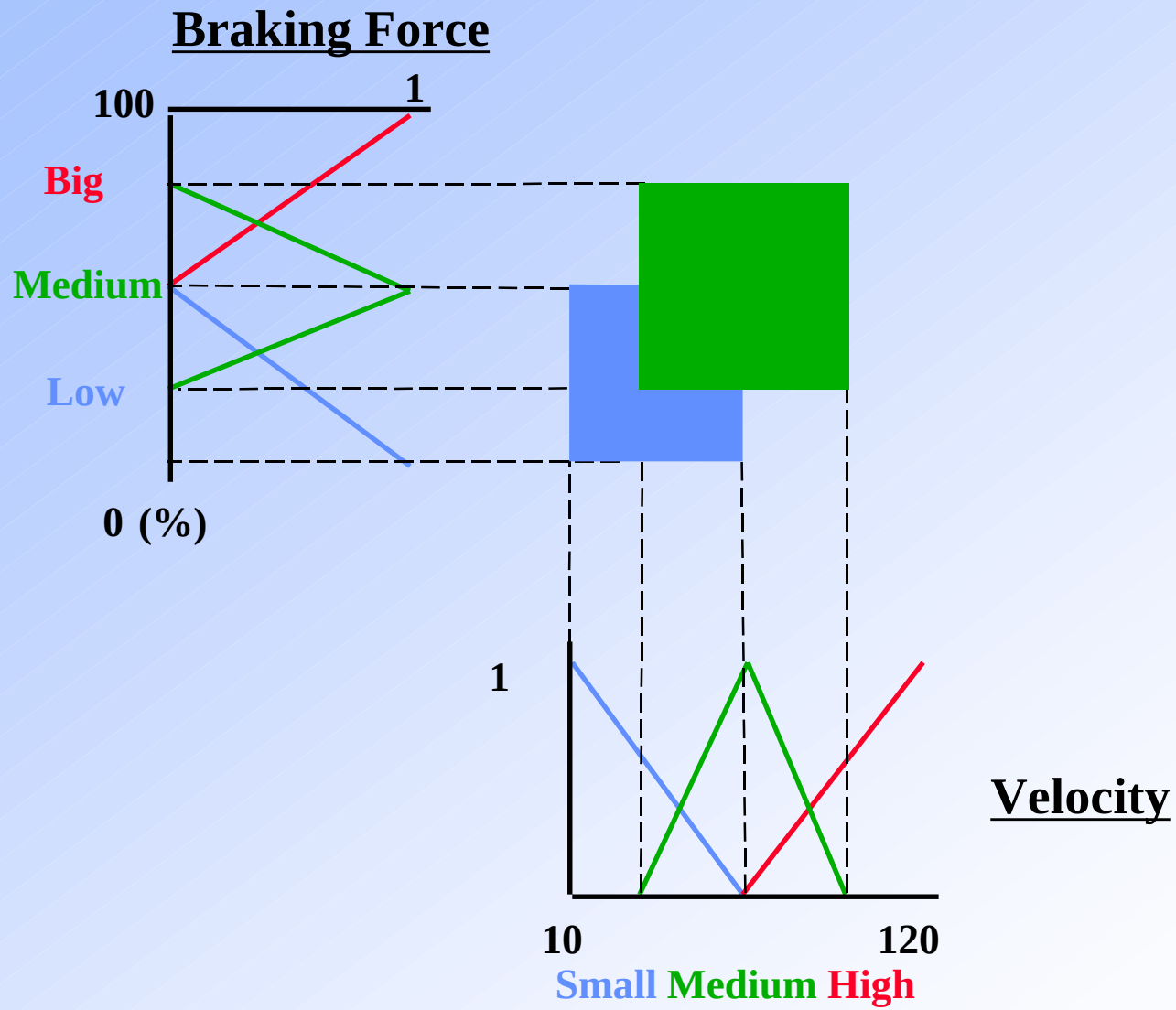

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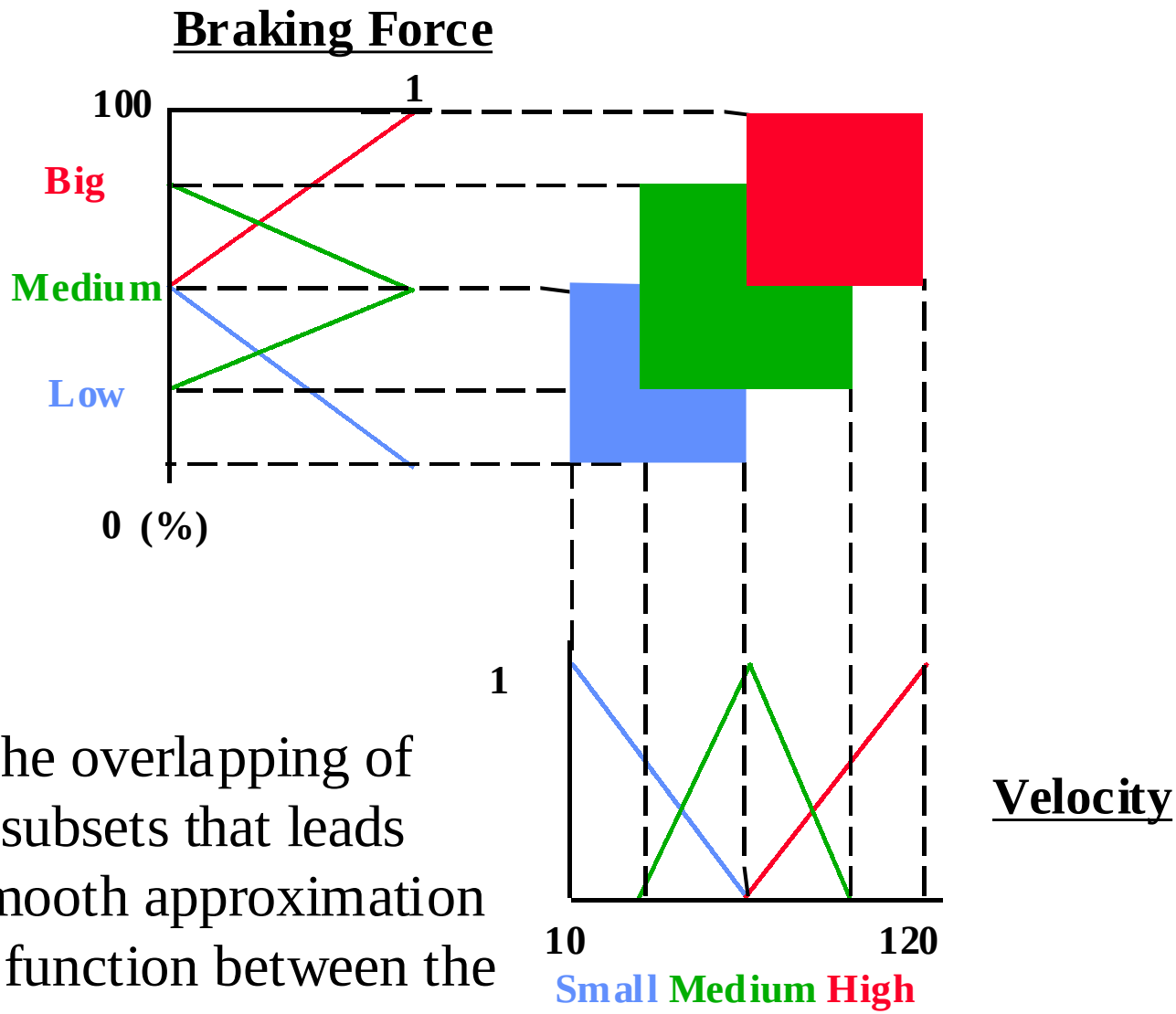
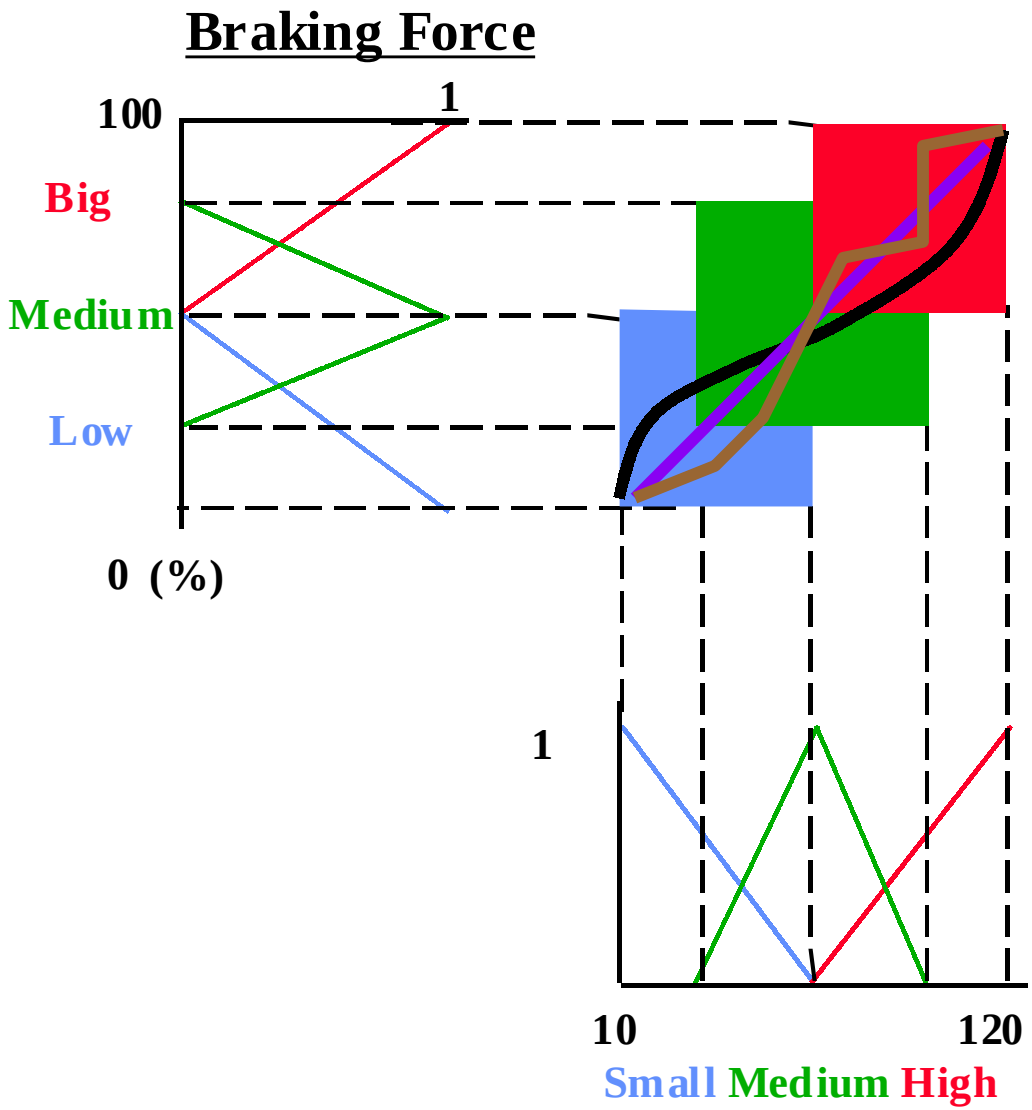


Note the overlapping of fuzzy subsets that leads to a smooth approximation of the function between the **Velocity** & **Braking Force**

The Fuzzy Patches Define the Function



Three possible dependencies between the v and B.

Each of us drives differently, and we all drive in different way each time.

FUNCTIONAL DEPENDENCE OF THE VARIABLES

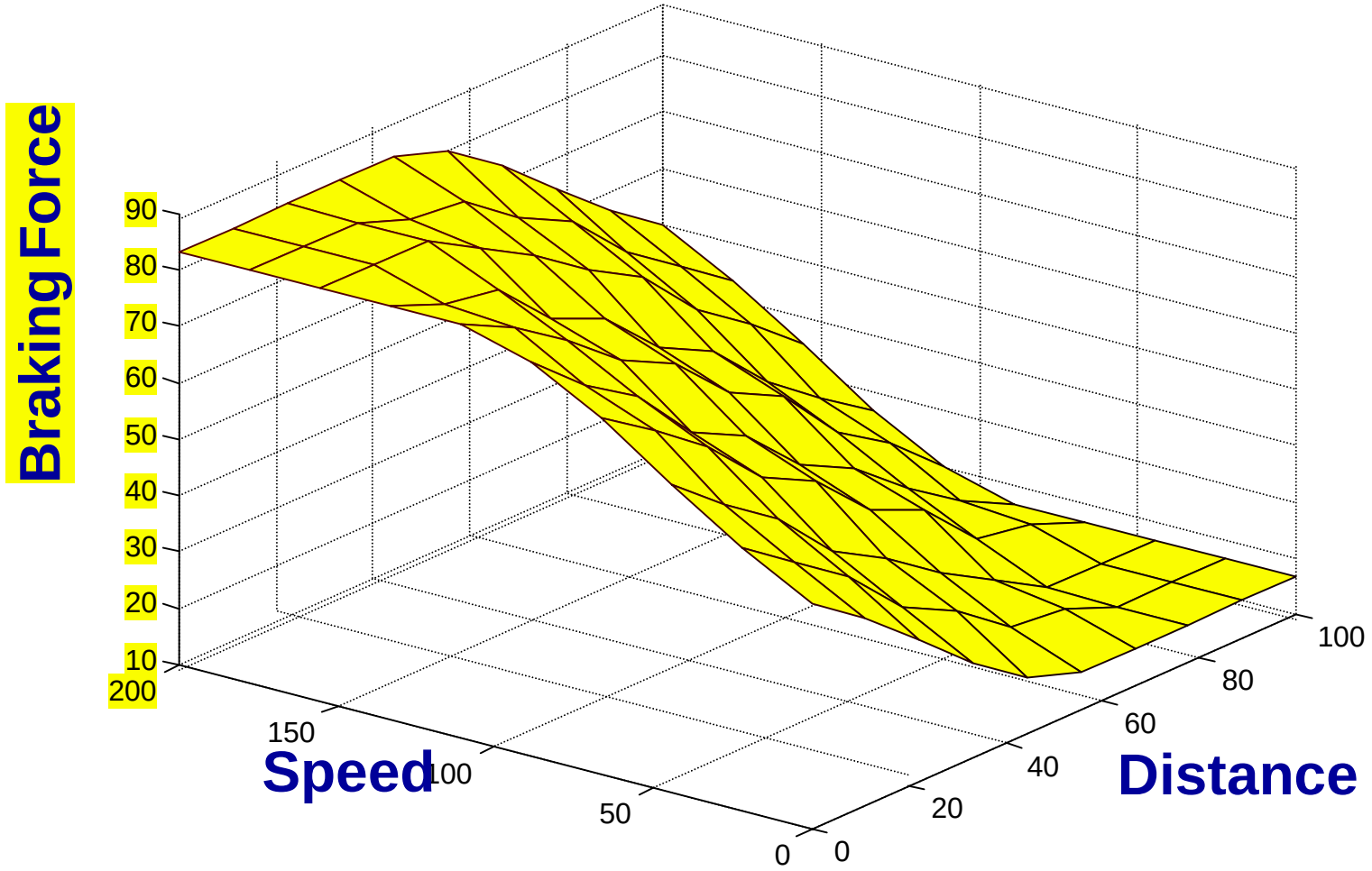
SURFACE OF KNOW LEDGE

V.K.

Fuzzy Control of the Distance Between Two Cars again

now, however,2 INPUTS: **D** and **v,** and 1 OUTPUT **B** ⇒ **visualization still possible.**

For more inputs everything remains same but no longer visualization.

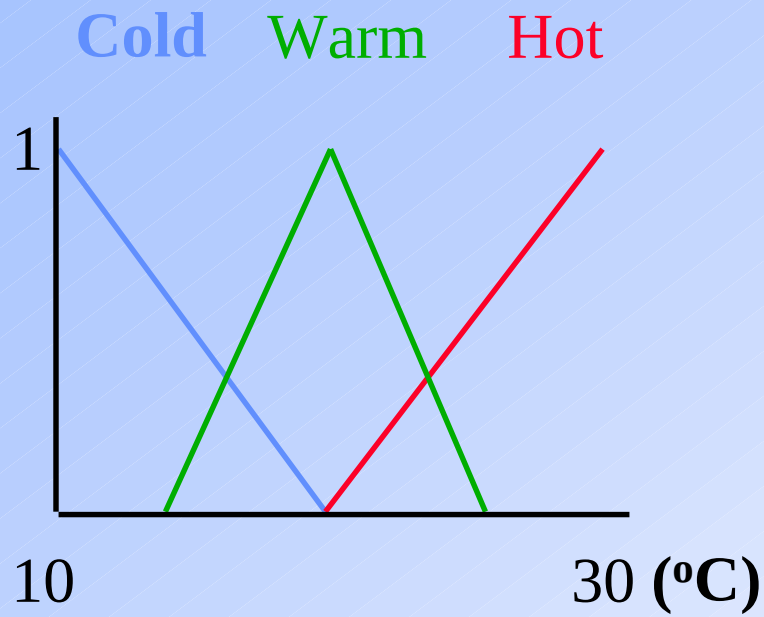


Second Example

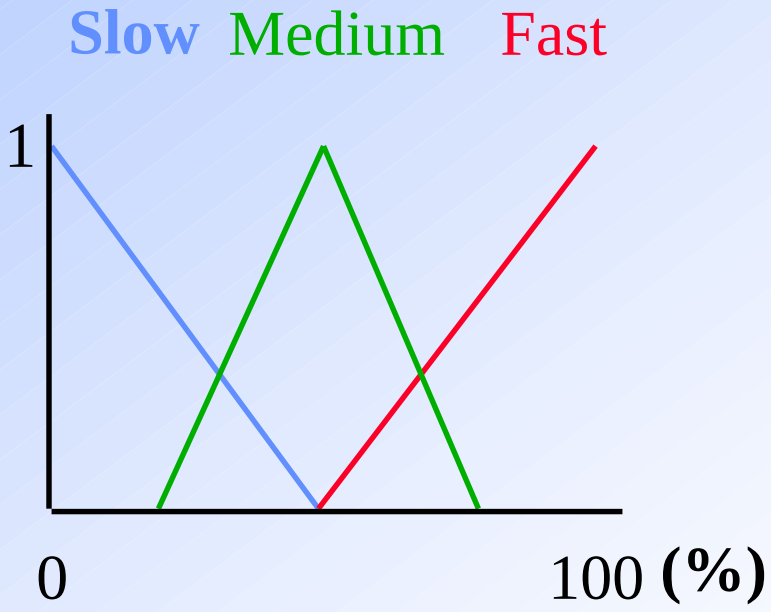
Fuzzy

Room Temperature Control

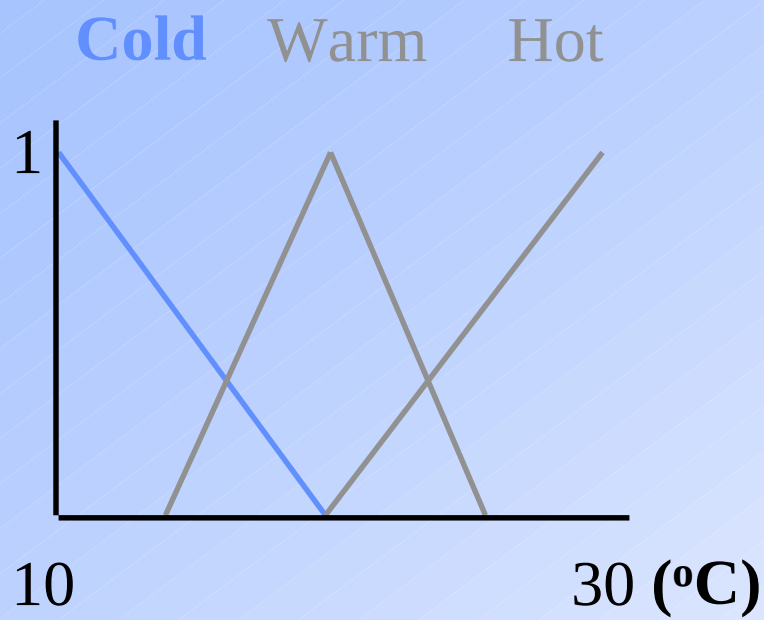
Room Temperature



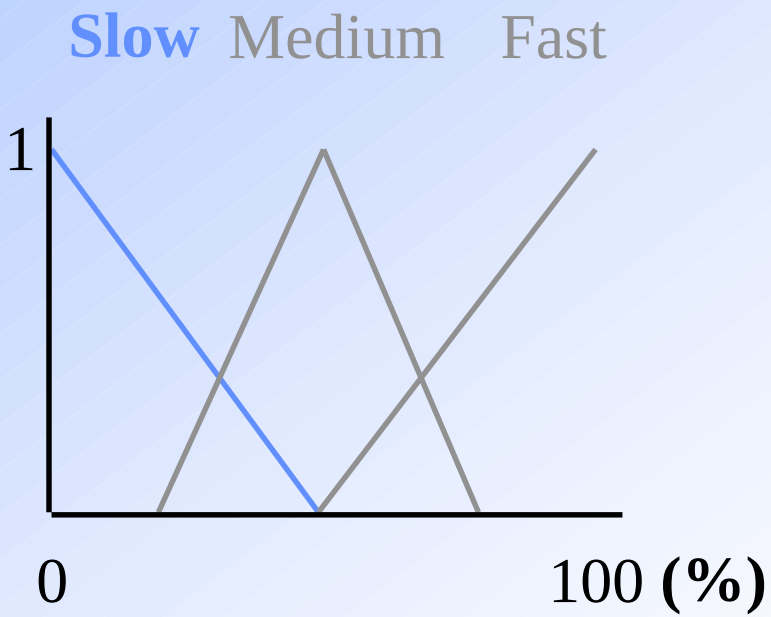
Fan Speed



Room Temperature

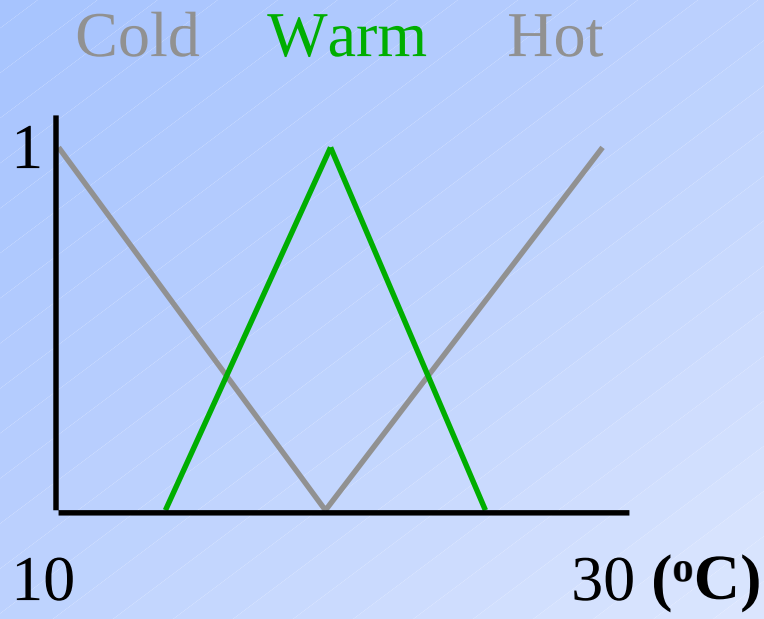


Fan Speed

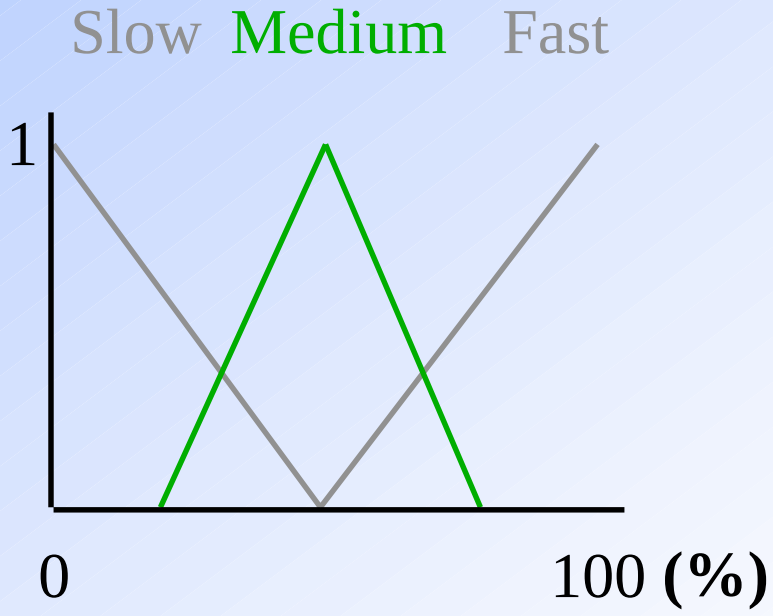


If Room Temperature is **Cold** then Fan Speed is **Slow**

Room Temperature



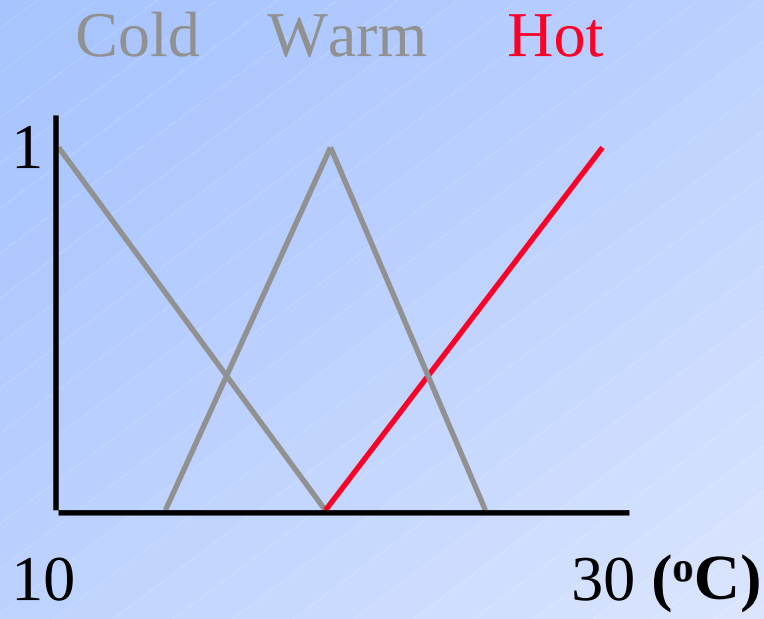
Fan Speed



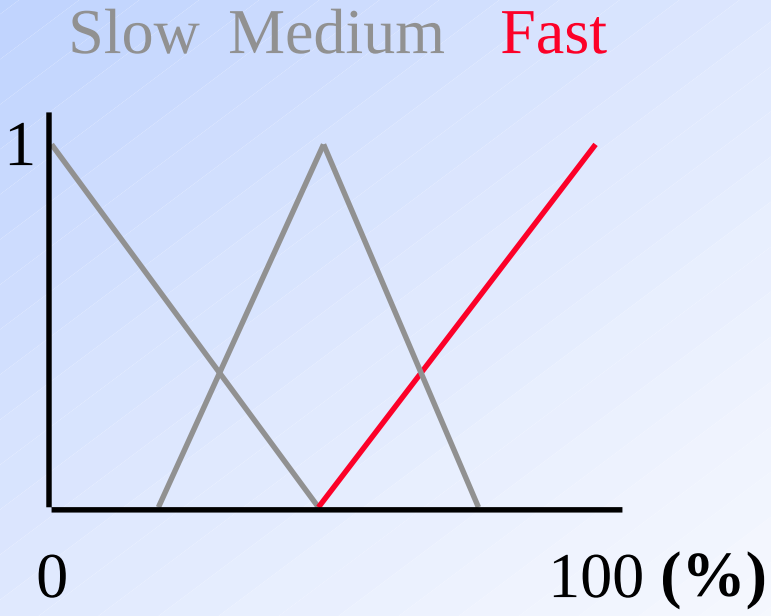
If Room Temperature is Cold then Fan Speed is Slow

If Room Temperature is **Warm** then Fan Speed is **Medium**

Room Temperature



Fan Speed

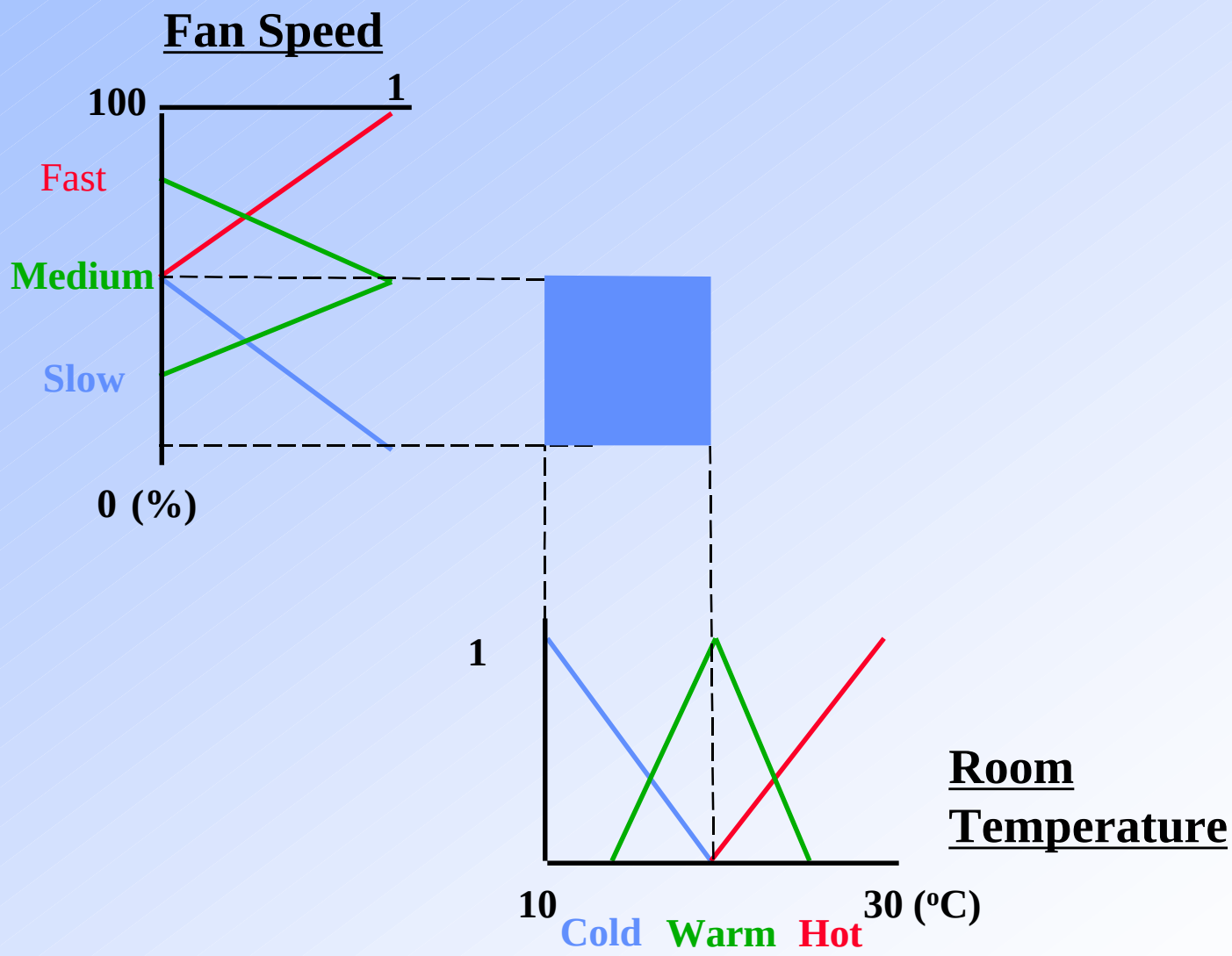


If Room Temperature is Cold then Fan Speed is Slow

If Room Temperature is Warm then Fan Speed is Medium

If Room Temperature is **Hot** then Fan Speed is **Fast**

The Fuzzy Patches



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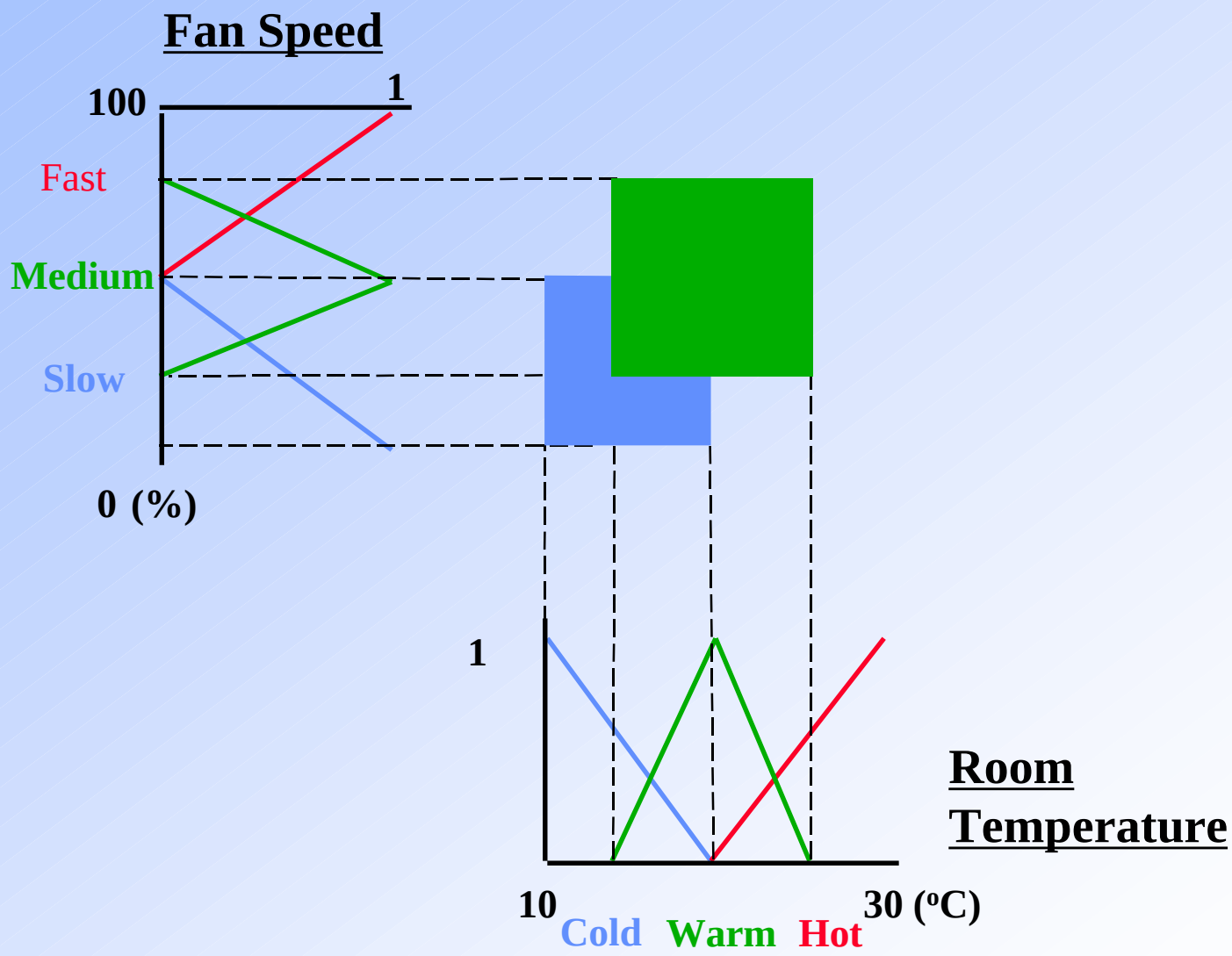

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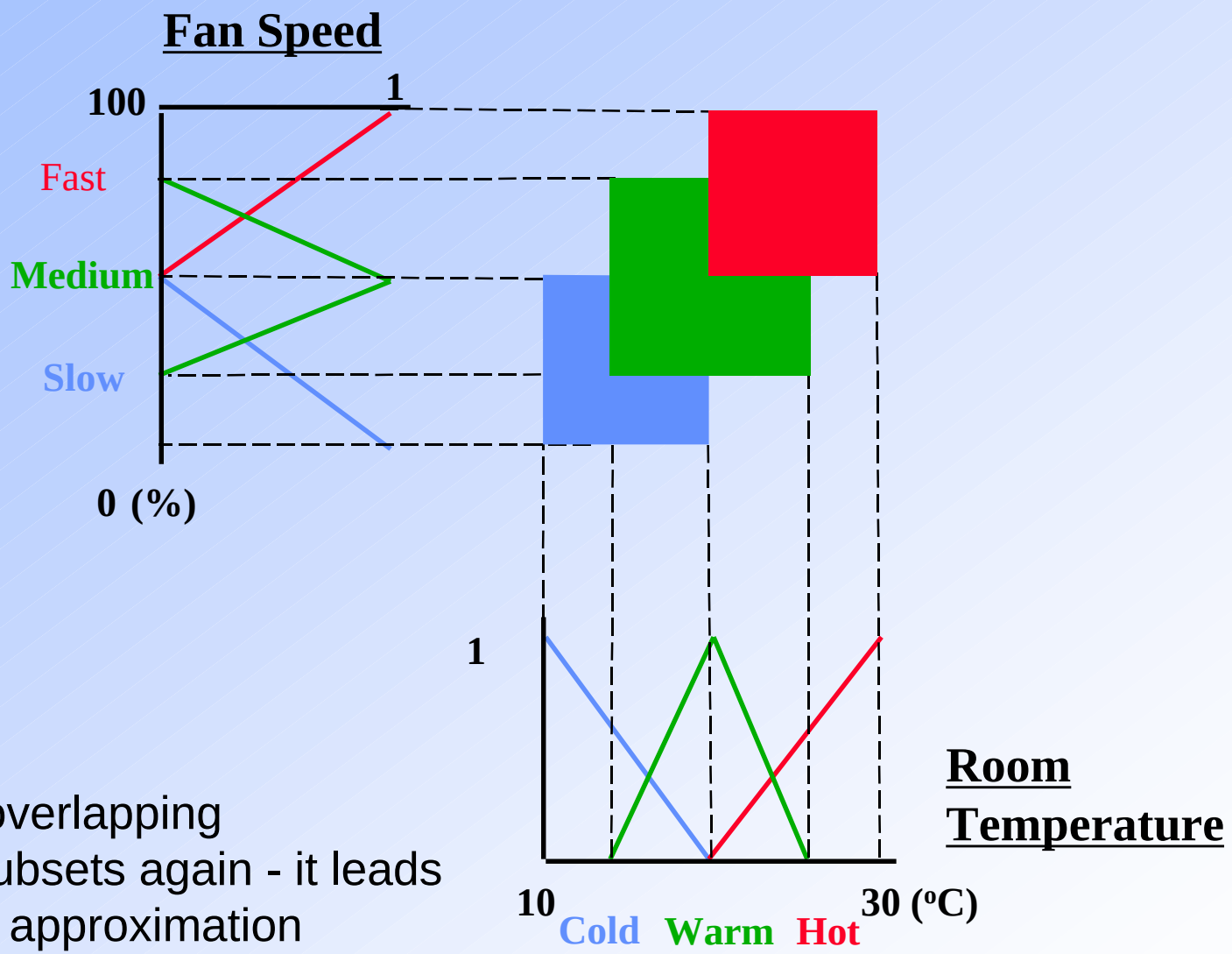
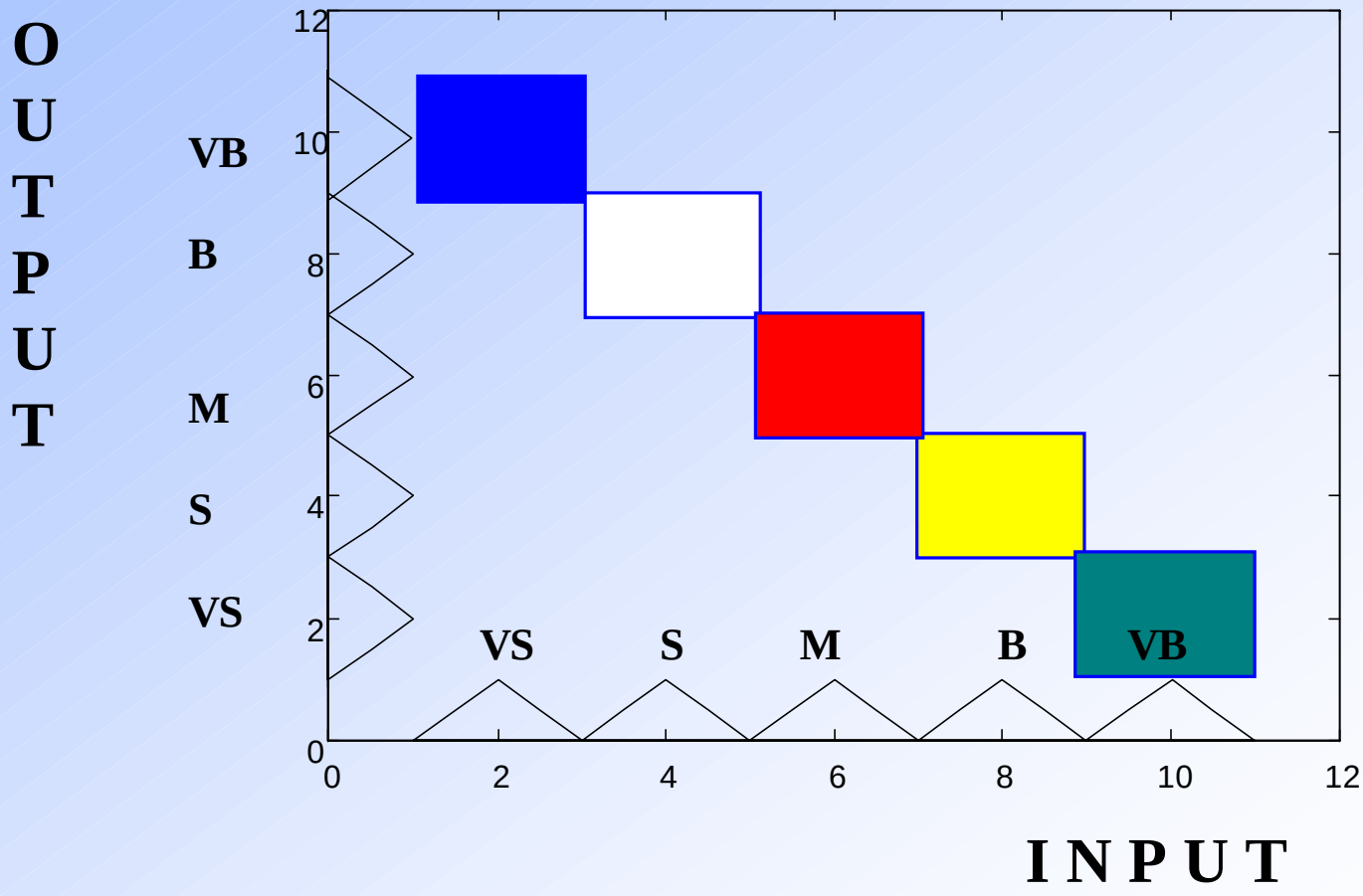


Note the overlapping
of fuzzy subsets again - it leads
to smooth approximation
of the function between the
Fan Speed & Temperature

There must be some overlapping of the input fuzzy subsets (membership or characteristic functions) if we want to obtain a smooth model



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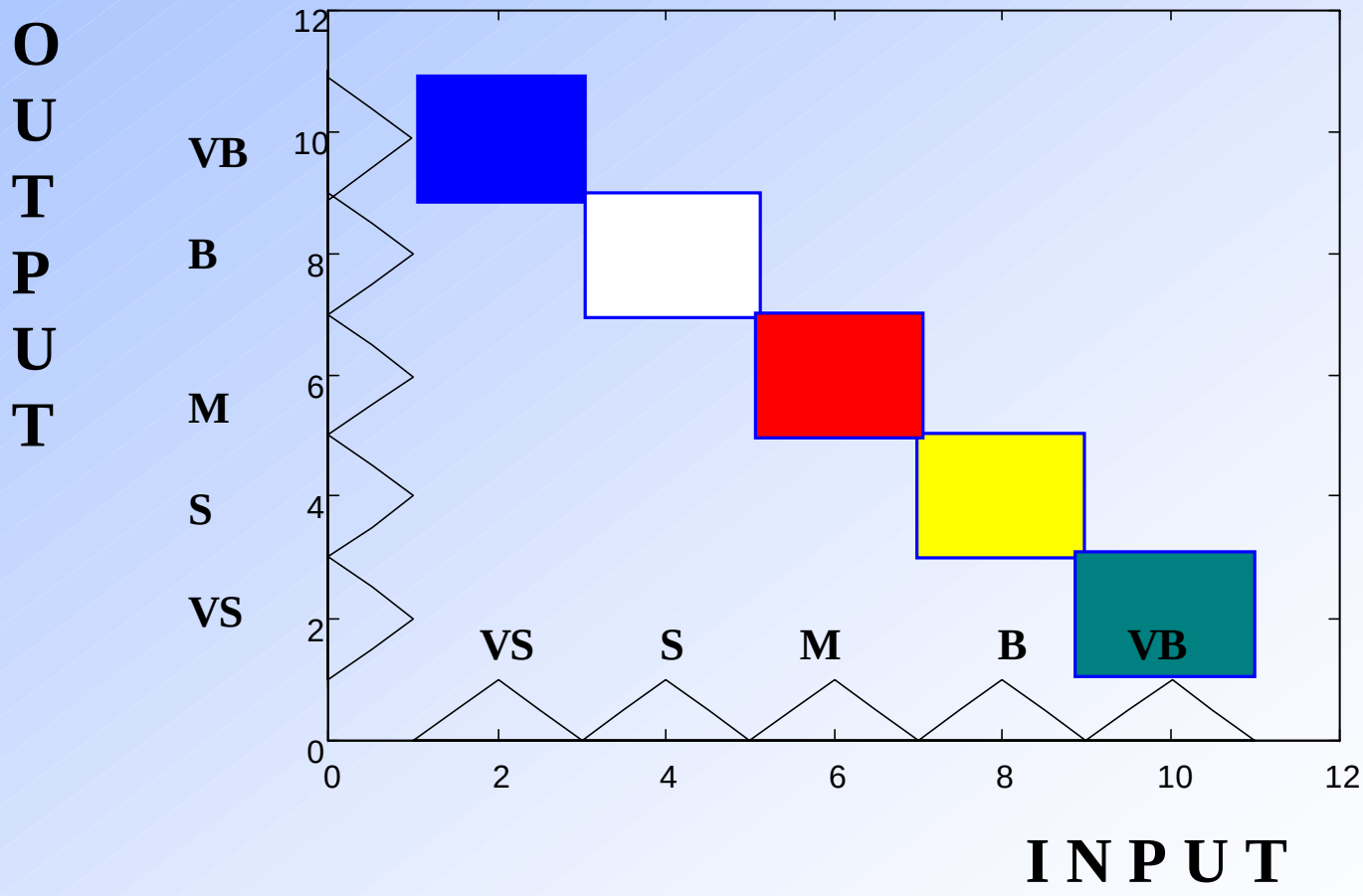
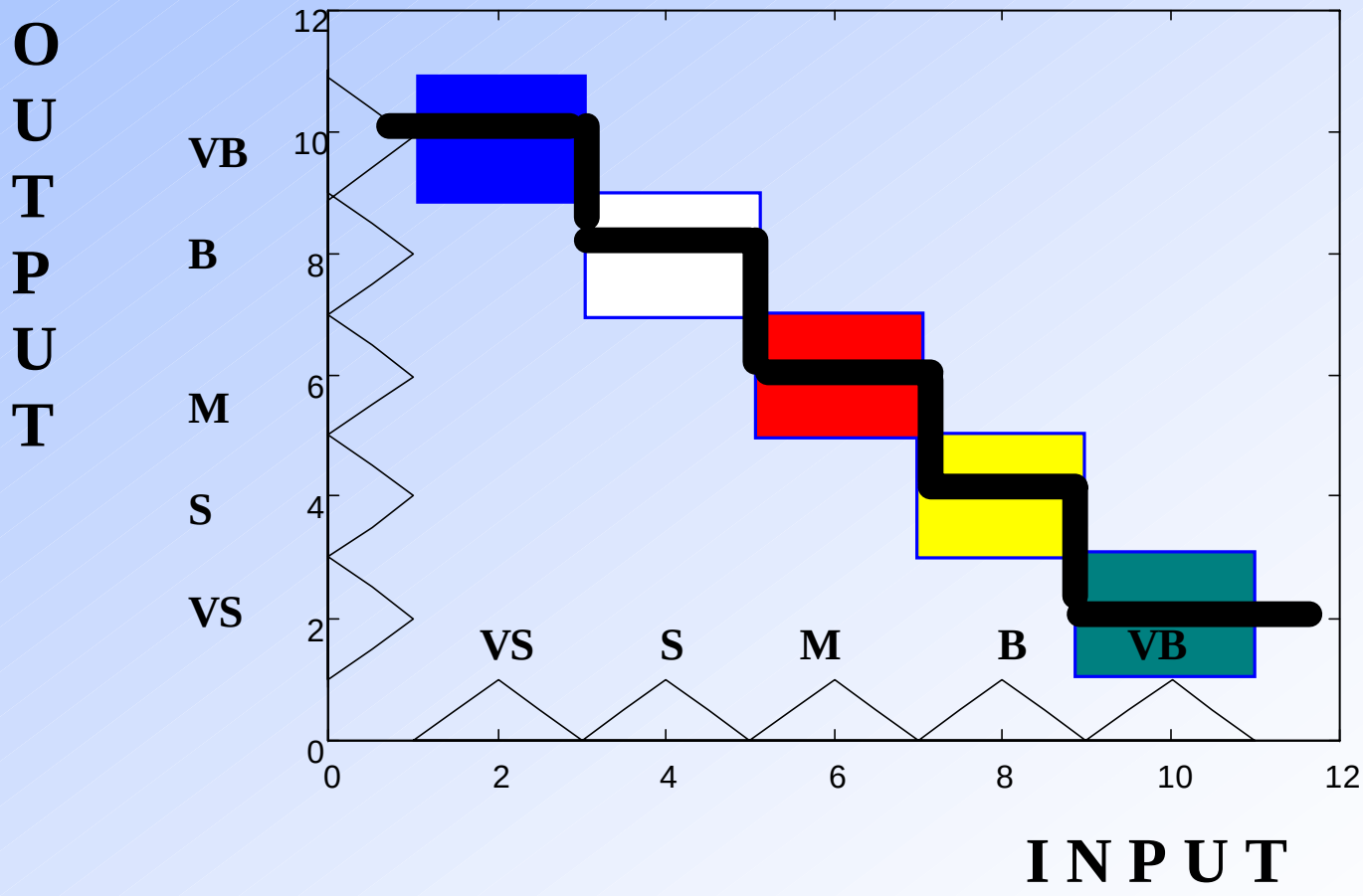


Because,

if there is no overlapping one obtains the stepwise function as shown next!

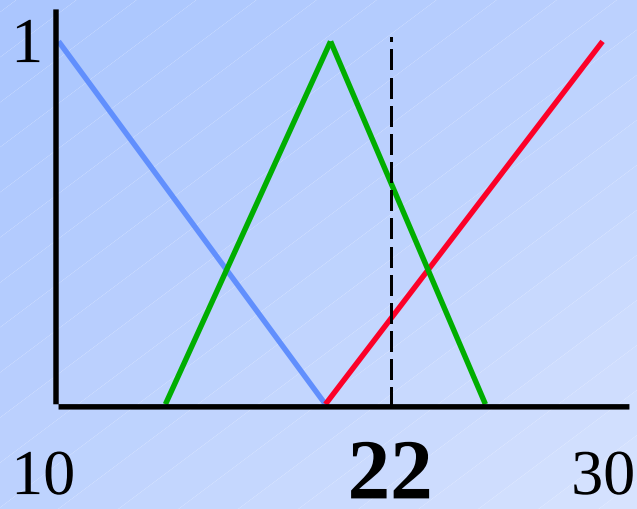
There must be some overlapping of the input fuzzy subsets (membership or characteristic functions) if we want to obtain a smooth model



Calculation of the output: Fuzzification, Inference and Defuzzifaction

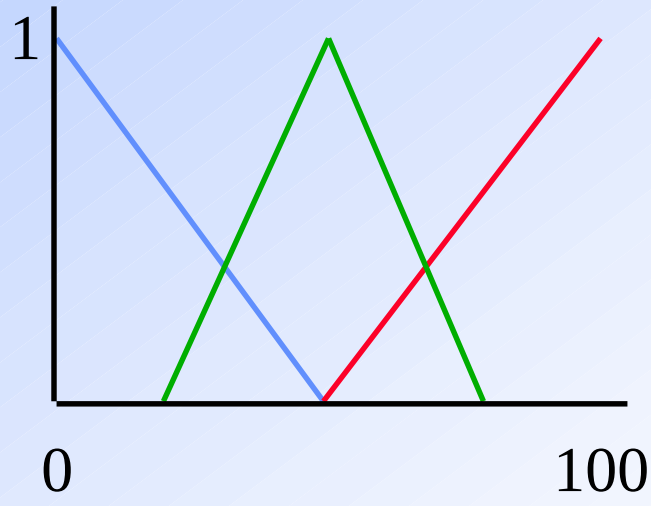
Room Temperature

Cold Warm Hot



Fan Speed

Slow Medium Fast



R1: If Room Temperature is Cold Then Fan Speed is Slow

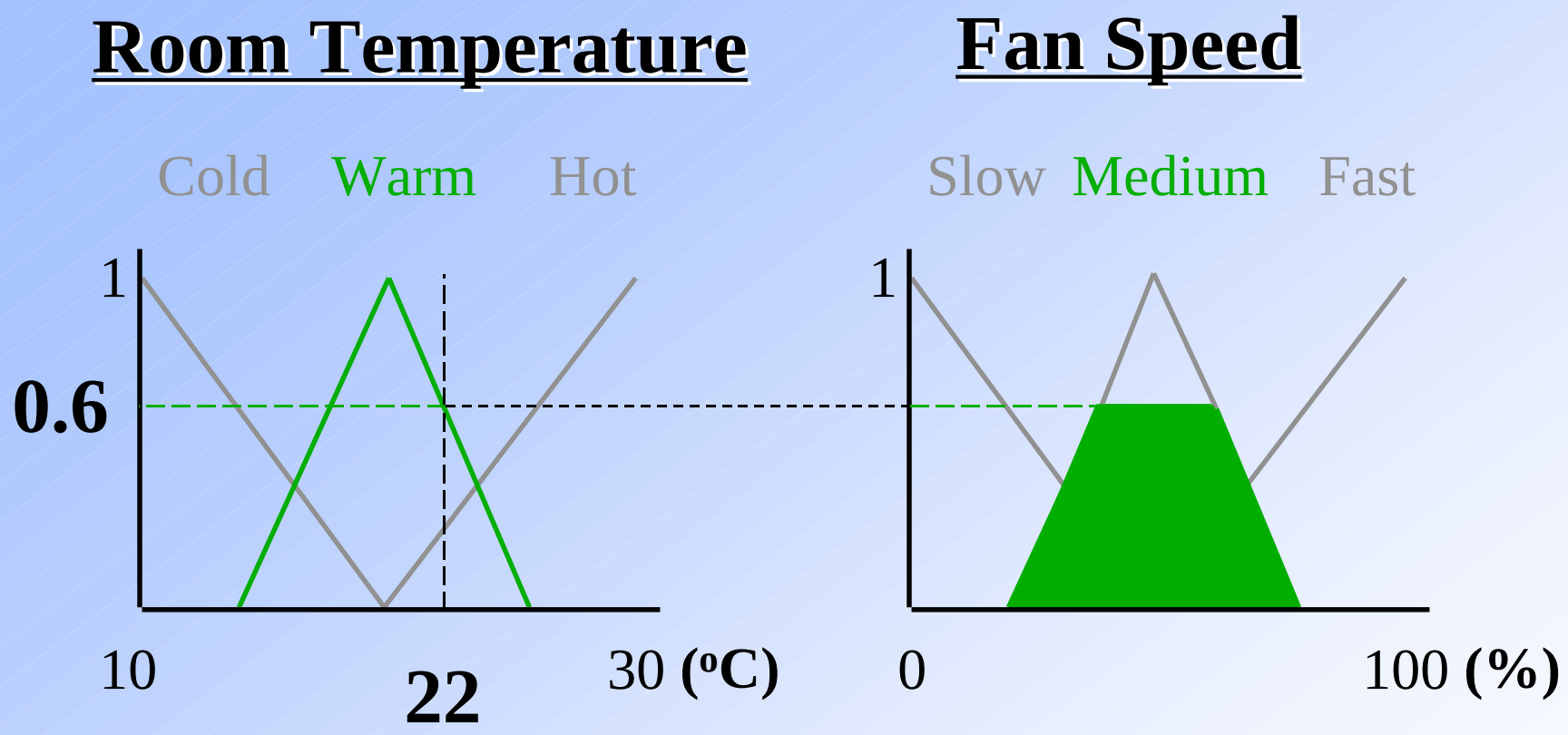
R2: If Room Temperature is Warm Then Fan Speed is Medium

R3: If Room Temperature is Hot Then Fan Speed is Fast

After the fuzzy modeling is done there is an operation phase: Calculate the Fan Speed when Room Temperature = 22 ºC .

NOTE! 22 ºC belongs to the Subsets Warm and Hot

Fuzzification and Inference

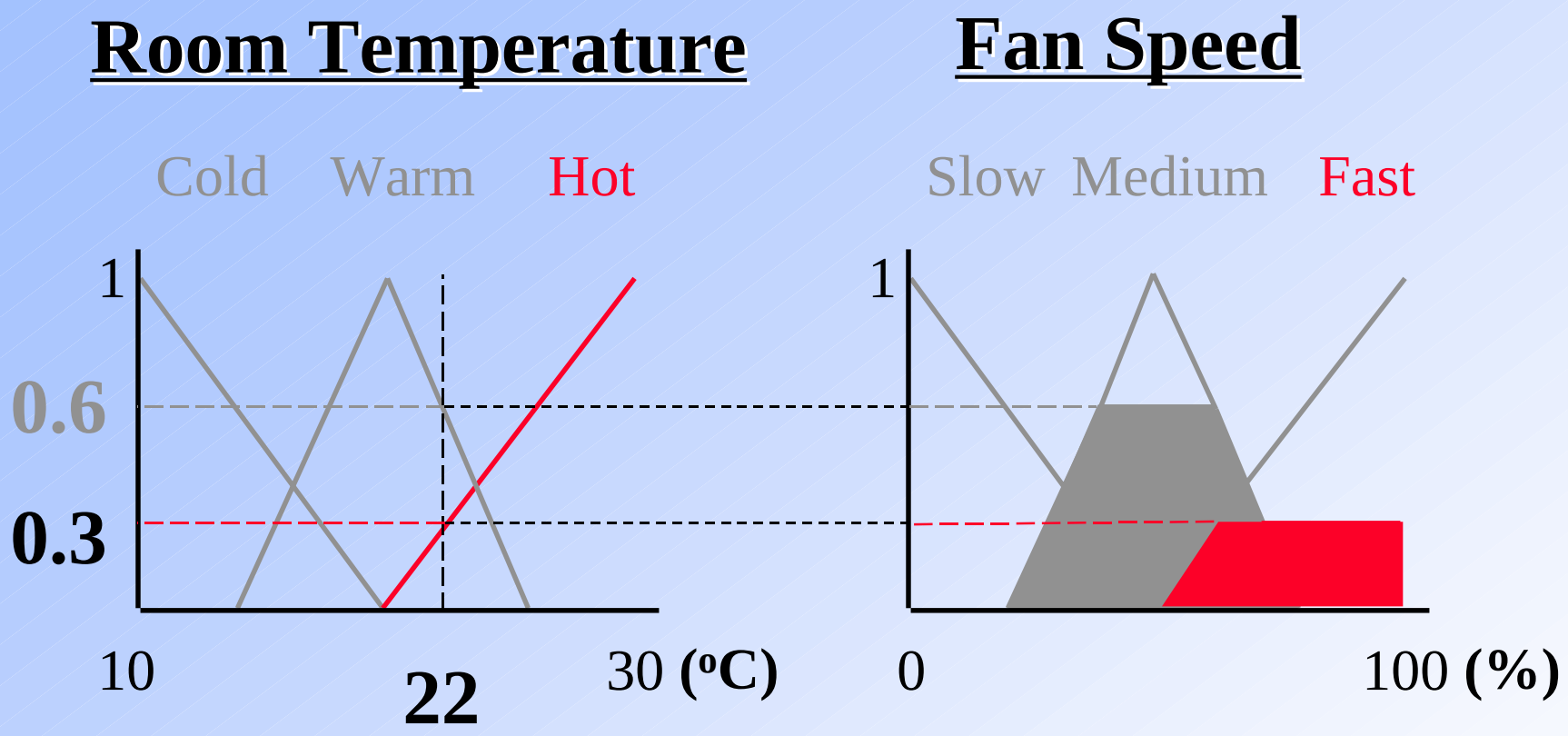


If Room Temperature is Cold Then Fan Speed is Slow

If Room Temperature is **Warm** Then Fan Speed is **Medium**

If Room Temperature is Hot Then Fan Speed is Fast

Fuzzification and Inference

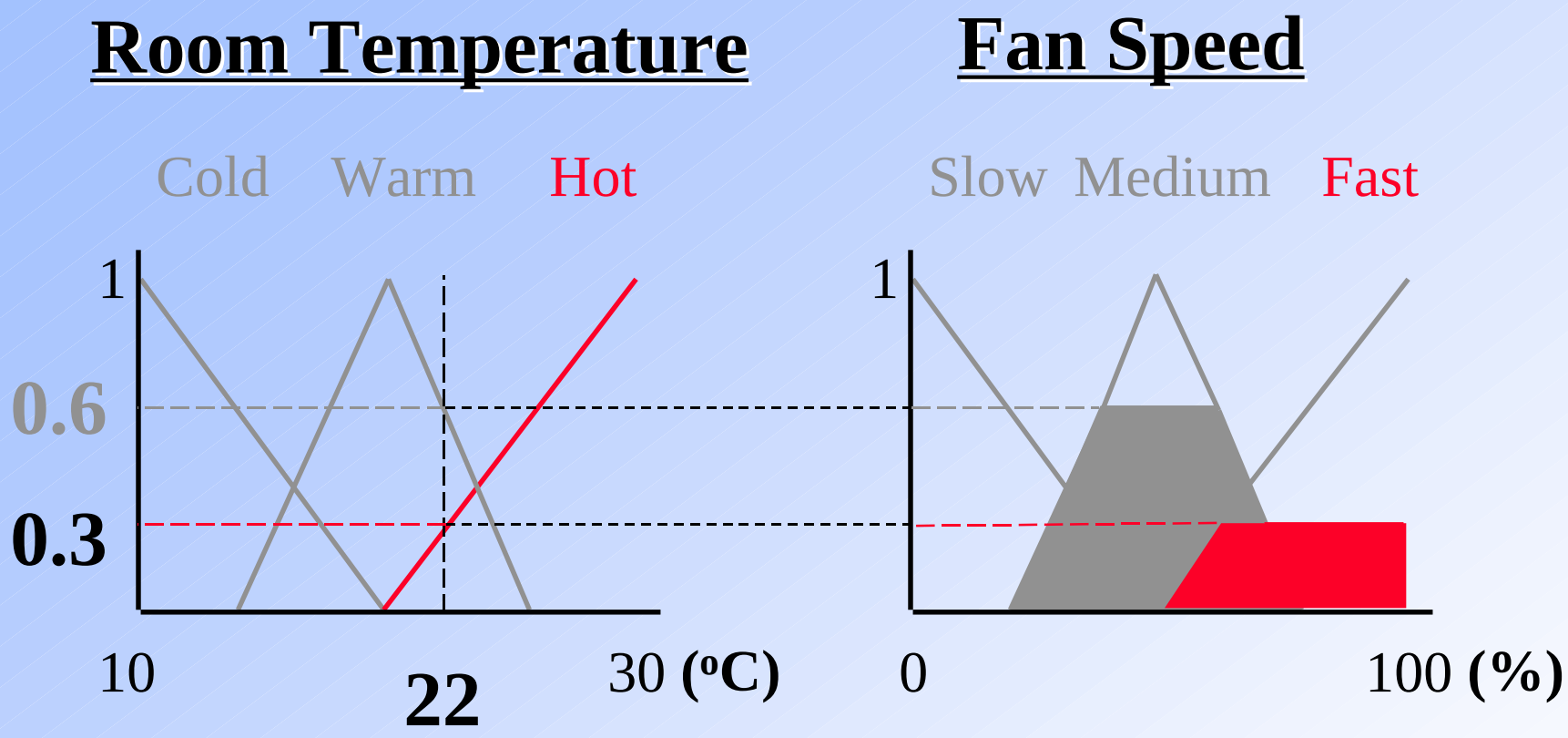


If Room Temperature is Cold then Fan Speed is Slow

If Room Temperature is Warm then Fan Speed is Medium

If Room Temperature is **Hot** then Fan Speed is **Fast**

Fuzzification and Inference



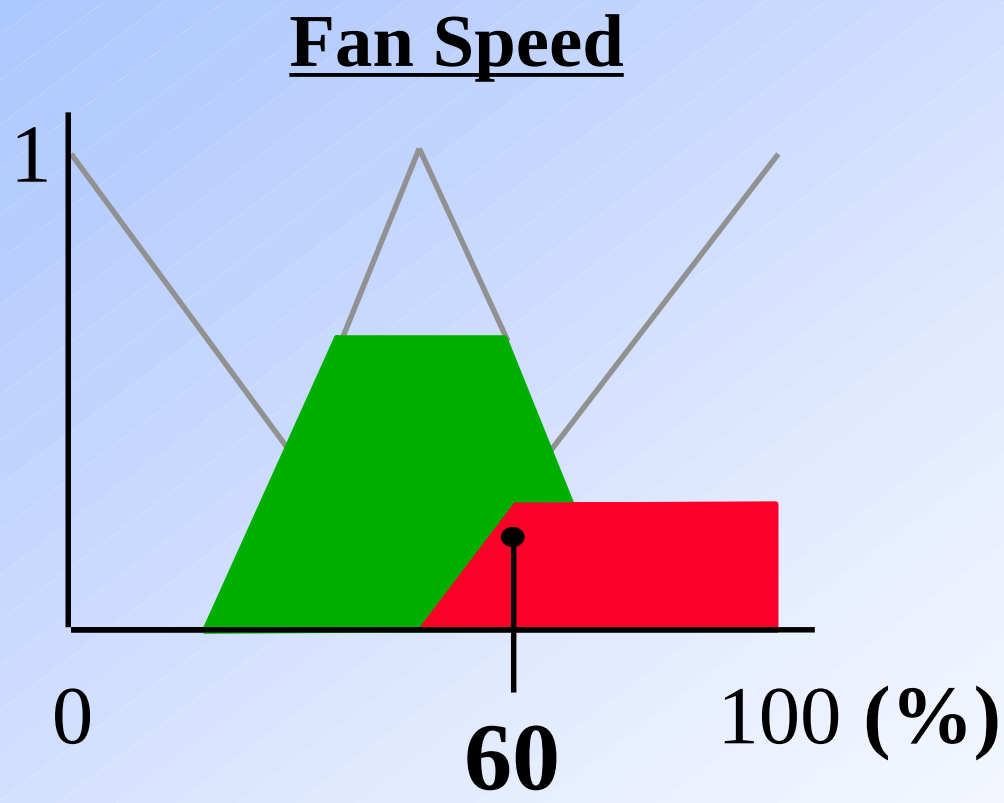
If Room Temperature is Cold then Fan Speed is Slow

If Room Temperature is Warm then Fan Speed is Medium

If Room Temperature is **Hot** then Fan Speed is **Fast**

THE QUESTION NOW IS: WHAT IS THE OUTPUT VALUE?

Defuzzification



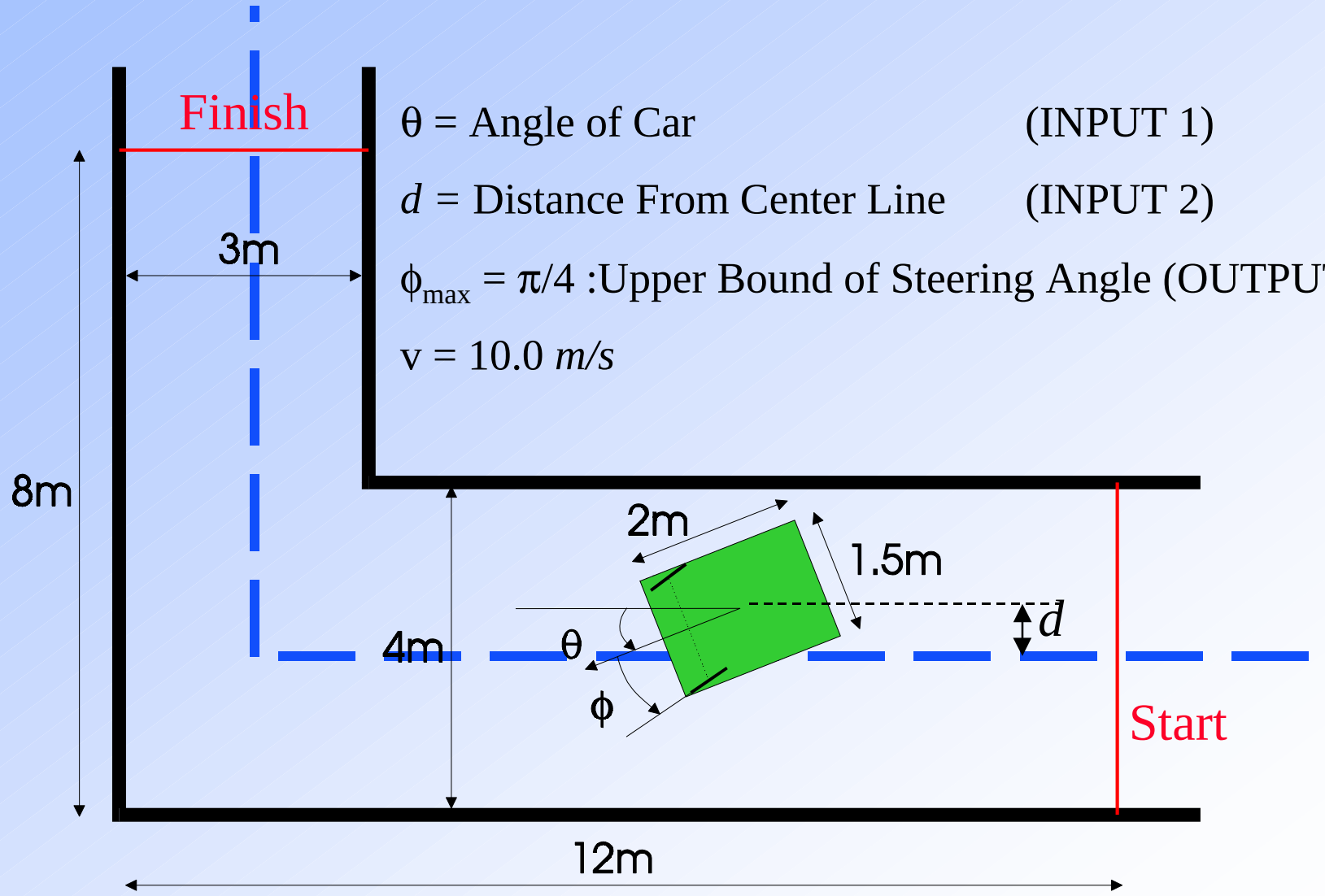
- **The Result of the Fuzzy Inference is a Fuzzy Subset Composed of**
  - **the Slices of Fan Speed: Medium (green) and Fast (red)**
- **How to Find a Crisp (for the real world application useful) Value?**
- **One out of several different methods SHOWN ABOVE is:**
  - **the Centre of Area Formula to Obtain a Crisp Output**

Example given in program LEARNSC from the book:

i) Vehicle Turning Problem

- Generic Fuzzy Logic Controller
  - Developed in Matlab
  - User Friendly
  - Multiple Inputs
- Many Other Commercial Applications Possible

Configuration of the Vehicle Turning Problem



θ = Angle of Car (INPUT 1)

d = Distance From Center Line (INPUT 2)

$\phi_{max} = \pi/4$:Upper Bound of Steering Angle (OUTPUT)

v = 10.0 *m/s*

CONCLUSIONS

- **Fuzzy Logic Can Be Implemented Wherever There is Structured Human Knowledge, Expertise, Heuristics, Experience.**
- **Fuzzy Logic is not needed** whenever **there is an analytical closed-form model** that, using a **reasonable number of equations**, can **solve the given problem in a reasonable time**, at the **reasonable costs** and with **higher accuracy**.
- POSSIBLE PROBLEMS:
- Finding **good (accountable)** expert,
- Right choice of the variables,
- Increasing the number of *inputs*, as well as the number of fuzzy subsets per input variable, the number of rules increases exponentially (**curse of dimensionality**)
- **Good news is**, that there are plenty of real life problems and situations that can be solved with small number of rules only.